SELF SATURATING MAGNETIC AMPLIFIERS

BY

DANIEL BARRY WILDER, JR.

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D.B. WILDER, JR.

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Daniel Barry Wilder, Jr. - 1414

Lieutenant Commander, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE

United States Naval Postgraduate School
Annapolis, Maryland
1950

This work is accepted as fulfilling
the thesis requirements for the degree of

MASTER OF SCIENCE

from the

United States Naval Postgraduate School.

PREFACE

The self saturating magnetic amplifier is one more application of the saturable core reactor. The general principles of the saturable core have been known for a long time. The writer has tried to apply them to the self saturating reactor.

Appreciation is expressed to all those of the General Engineering and Consulting Laboratories, General Electric Company, Schenectady, N.Y. who have cooperated in giving advice and assistance. Among them the writer feels particularly indebted to H.M. Ogle, Ray E. Morgan, and Hugh Schirk. Also the writer expresses his deepest appreciation to his staff advisor Dr. W.M. Bauer for his assistance and guidance.

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SYMBOLS

CHAPTER TWO

SYMBOL	DESCRIPTION
A	Cross section area of core in square inches.
$B_{max.}$	Maximum value of flux density contributed by the supply voltage.
B	Flux density in core
B_b	Flux density required to saturate the core.
B_o	Flux density of the core with zero anode current.
C_1 & C_2	Constants dependent upon magnetization curve
f	Frequency of the supply voltage.
G	Overall or mean power gain from minimum to maximum output.
H_o	Constant dependent upon the initial and maximum value of the core flux density.
H_n	Constant for each harmonic whose value is computed with the aid of the table at the end of Appendix B.
I_L	Average value of the load current for two anode coils.
$I_{L(max)}$	Average value of load current at the point on I_c vs. I_L curve where doubling the I_c increases the load current approximately 25%.
I_c	Average value of current in the control winding.
I_{cm}	Control winding current for $I_{L(max)}$ in the load

SYMBOLS

CHAPTER TWO
(Cont.)

SYMBOL	DESCRIPTION
I_m	Magnetization current.
I_{ave}	Rectified average value of the anode current.
i_1	Instantaneous value of the control current.
i_2	Instantaneous value of current in anode #1.
i_3	Instantaneous value of current in anode #2.
k'	Constant
k_i	Current constant of core material
k_t	Constant of core material to determine time for the reactor to go from minimum load to $I_{L(max)}$.
k_v	Voltage constant of core material.
L	Inductance in henries.
ℓ	Length of ac flux path in inches.
N, N_2, N_{ac}	Number of turns on anode winding.
N_c, N_{dc}	Number of turns on control winding.
n	Number
R_c	Resistance of control winding circuit.
R_1	One half coil resistance of control winding.
R_r	Forward resistance of rectifier.
R_2	Resistance of one anode coil.
R_z	Anode coils resistance reflected into the control winding.
r	Ratio of the dc flux to the maximum ac flux.

SYMBOLS

CHAPTER TWO
(Cont.)

SYMBOL	DESCRIPTION
T	Time in seconds for the load current to increase from minimum value to $I_{L(max)}$.
t_c	Time constant based on linear circuits.
U	Total flux.
U'_o	Total flux produced by control current.
U_o	Total flux produced by a control current in one anode winding.
U_m	Maximum value of flux contributed by supply voltage.
V_s	RMS value of the supply voltage.
V_L	AVE value of the rectified load voltage.
W_L	Load watts.
θ_f	Angle of the supply frequency when the core saturates.
β	Feedback factor.

SYMBOLS

APPENDIX A

SYMBOLS	DESCRIPTION
A	Cross section area in square inches
B	Magnetic flux density in maxwells per square inch.
B_m	Maximum value of the magnetic flux density due to the supply voltage.
B_f	Flux density of core at time of saturation.
B_o	Flux density of core when no current is flowing in the anode winding.
E_f	Defined by equation 19a in Appendix (A).
E_m	Maximum value of the supply voltage.
e_s	Instantaneous value of supply voltage.
e_L	Average value of load voltage.
e_r	Rectifier voltage drop.
H	Magnetizing force per inch, in gilberts per inch.
i_2	Instantaneous current in one anode winding.
f	Length of magnetic path in inches.
R	Resistance in the anode circuit less the rectifier resistance.
t	Time in seconds.
t_f	Time from zero supply voltage until core saturates.
θ_f	Angle of core saturation, measured from zero supply voltage.

SYMBOLS

APPENDIX A
(Cont.)

SYMBOLS	DESCRIPTION
U or φ	Value of flux in maxwells.
w	Angular frequency in radians.

SYMBOLS

APPENDIX B

SYMBOLS	DESCRIPTION
A	Cross section area in square inches.
B	Magnetic flux density in kilolines per square inch.
B_{1c}	Magnetic flux density in core one in kilolines per square inch.
B_{2c}	Magnetic flux density in core two in kilolines per square inch.
B_o	Average value of flux density in each core.
B_m	Peak value of alternating flux density in kilolines per square inch.
E_{dc}	Value of constant control voltage.
E_m	Peak value of applied sinusoidal voltage.
H	Magnetic field intensity in ampere turns per inch.
H_{1c}	Magnetic field intensity in core #1 in kilolines per square inch.
H_{2c}	Magnetic field intensity in core #2 in kilolines per square inch.
I_1	Average value of the current in the control winding.
k'	Constant $\frac{N_1}{N_2} \quad \frac{(R_2 + R_r)}{R_1}$
R_r	Constant forward resistance of rectifier.
U	Constant used in representing B-H Curve by hyperbolic sine, in ampere turns per inch.

SYMBOLS

APPENDIX B
(Cont.)

SYMBOL	DESCRIPTION
ℓ	Length of magnetic circuit in inches.
t	Time in seconds.
u	Constant used in representing B-H magnetization curve, inches^2 / kilolines.
v	Constant used in representing B-H magnetization curve, $\frac{\text{ampere turns - inches}}{\text{kilolines}}$
σ	Time in radians that i_3 begins to conduct.
φ 1c	Magnetic flux in core #1, in webers.
φ 2c	Magnetic flux in core #2, in webers.
	Time derivitive of flux, webers/seconds
w	Angular frequency, in radians per second.
I	Modified Bessel function of first kind of order n
i_1	Instantaneous current in the control winding.
i_2	Instantaneous current in #1 anode winding.
i_3	Instantaneous current in #2 anode winding.

SYMBOLS

APPENDIX C

SYMBOL	DESCRIPTION
A	Cross section area.
B_m	Maximum value of the flux density in the anode core.
C_1, C_2	Constants of magnetization curve.
E'_{rms}	Max. RMS value of supply voltage for which the curves are used.
E_{rms}	Root mean square of the supply voltage.
E_{rpu}	E_{rms}/E'_{rms}.
I_{ave}	Average value of the anode current less the straight line portion as defined by equation #41.
$I_{T(ave)}$	Average value of the anode current.
I_{apu}	I_{ave}/I_{rms}.
I_2	RMS value of the anode current.
I_n	Peak value of the (n)th harmonic current.
I_{rms}	RMS value of the anode current minus the straight line portion as defined by equation #18.
I'_{rms}	Current corresponding to E'_{rms}.
I_{rpu}	I_{rms}/I'_{rms}.
i_2	Instantaneous value of the anode current.
r	U_o/U_m.
U	Instantaneous value of the flux linkage of one anode coil.

APPENDIX C
(Cont.)

SYMBOL	DESCRIPTION
U_m	Peak value of ac flux linkage due to supply voltage.
U_o	dc flux linkage of one anode winding.
U'_o	Total dc flux in webers.

INTRODUCTION

Because the cycle of events inside a magnetic core is very difficult to measure, attempts at an explanation of saturable core reactors are by necessity based on simplifying assumptions. The discussions of this paper are made after several such assumptions.

The results are helpful in predicting the operating characteristics of the self saturated magnetic amplifier. The effects of distributed capacity, hysteresis, and eddy currents which are omitted by the simplifying assumptions, are generally handled as a special problem of the particular application. There is not enough general information available at present to include all these factors.

The solutions as presented are best suited for low frequency power supplies because of the simplifying assumptions.

These comments are made in the introduction to caution the reader that these are not general solutions and are good only as long as the approximations are valid.

CHAPTER ONE

A. DEFINITIONS

The self saturated magnetic amplifier is an adaptation of the saturable core reactor to a magnetic controlled circuit with considerable improvement in power gain as compared to the saturable reactor alone. For this reason it is probably the most important of the present magnetic amplifiers.

B. COMPARISON AND SIMILARITIES

One of the best ways for the electronic engineer to think of a saturable core reactor is in terms of a comparable vacuum tube circuit. A similar vacuum tube circuit is the grid controlled gas tube rectifier. The anode or ac winding of the magnetic amplifier presents a high impedance to the flow of current in this winding until such time as the core saturates. The time at which the supply voltage saturates the core can be controlled by the current in the control or signal winding, this action being similar to the control grid of a gas tube rectifier.

Once the core saturates, the instantaneous current in the anode winding is limited only by the circuit resistance. This compares with loss of grid control once the gas tube fires. To a first approximation the anode winding continues to conduct until the direction of the applied potential is reversed.

C. OPERATION

A cycle of operation of the circuit labelled 1b will next be explained. It consists of two magnetic cores each

with an anode winding. The control or signal winding is common to both cores. This circuit is commonly referred to as single ended, since like a triode, the anode current can change only in magnitude and not in direction. Assume the current in the signal winding to be zero, and further refer to one of the anode windings. Next, refer to Figure #3. NI on the horizontal axis represents the ampere turns in the one core due to current in the anode winding. Under steady state conditions there will be a residual flux density of magnitude "A" when the current in this anode has reached zero. The rectifier in series with the anode prevents the current reversing in the coil during the next half cycle and the magnetic flux density remains at "A".

During the half cycles of the supply voltage that are of the correct polarity for current to flow in the anode winding, the flux density should increase from "A" to "B" in approximately 120 electrical degrees of the supply frequency. (This is a still under the condition of zero current in the control winding.) The knee of the curve or point "B" is the point at which rapid buildup of anode current begins. A typical current waveform is shown in Figure #4.

This saturation or firing usually occurs in a very few electrical degrees of the supply frequency "E_s". The final instantaneous value of the anode current is limited by the impedance of the load, resistance of the anode winding, the air core inductance of the anode coil,

and the instantaneous value of the supply voltage "E_s".

The 120 electrical degrees of the supply voltage for core saturation is chosen for the no signal current conditions. Best control of the load current by the signal current is thus obtained. A typical output current vs. control current is shown in Figure #5. The control current is considered positive when it reduces the time required for the core to saturate. Reducing this time of saturation increases the current to the load contributed by each anode.

The current in the control winding can be thought of as shifting the hysteresis loop to the right or left in Figure #3. Or again, point "A" may be considered as moving along the upper curve of the hysteresis loop with zero anode current. Either viewpoint shows that the time of core saturation can be controlled by the current in the control winding.

D. CIRCUIT ARRANGEMENT

The two anode coils are so connected with the rectifiers that one saturates during each half cycle of the supply frequency. If the point "A" is moved toward "B" in one core during its half cycle of operation, then the point "A" must also move toward "B" in the other core during the next half cycle of operation. If the anode windings are wound in the same direction on the two cores, then they must be connected oppositely. This connection is indicated on some of the prints by calling the start "s" and the finish "f".

Figure "1a" and "1b" are similar as far as the magnetic circuit is concerned. The load current in Figure "1a" is ac and in "1b" is pulsed dc. Figure #2 is an example of a push pull circuit. Its connections are more involved and will not be covered here.

SELF SATURATING MAGNETIC AMPLIFIERS

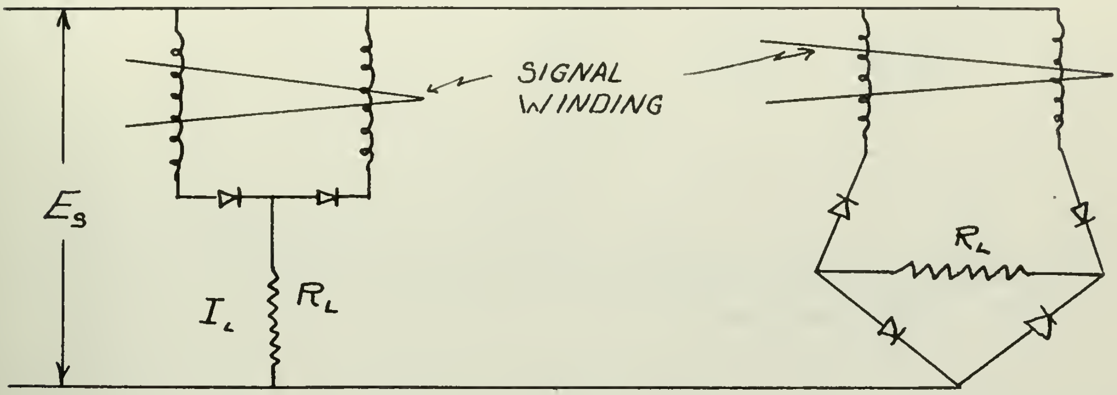


FIG. 1a FIG 1b

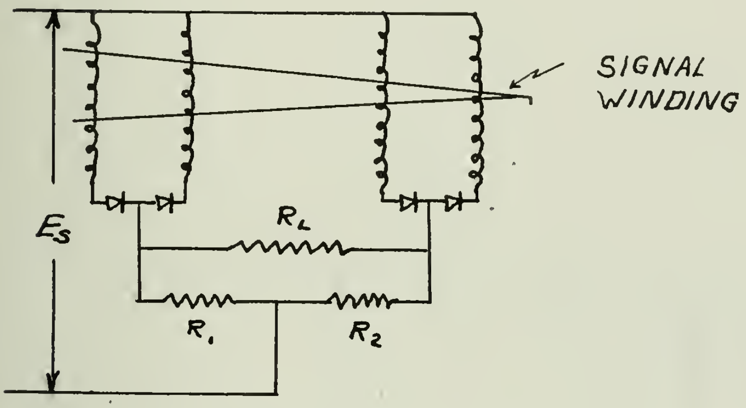


FIG 2

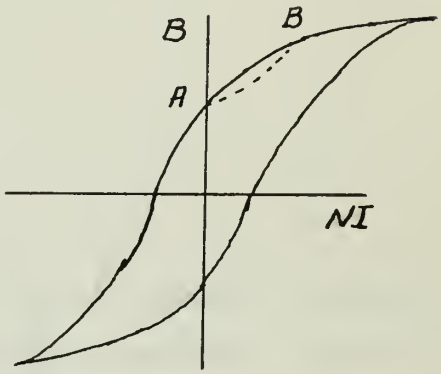
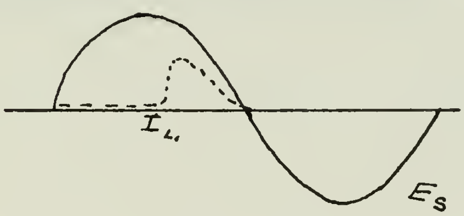


FIG 3

MAGNETIC CURVE WITH ZERO CONTROL CURRENT



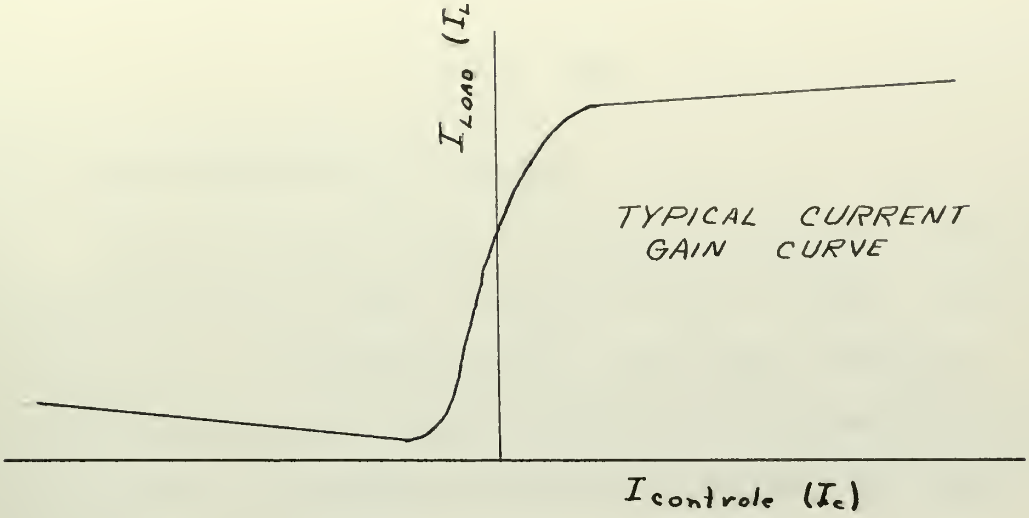


FIG 5

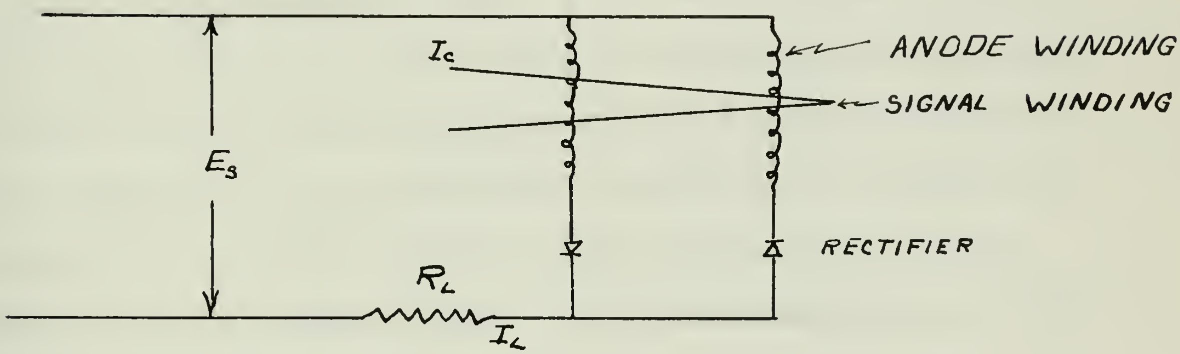


FIG 6

PARALLEL CONNECTED
MAGNETIC AMPLIFIER

CHAPTER TWO

A. CHARACTERISTIC AND DESIGN

Since the operation of the self saturated amplifier is different for high and low driving sources, the operation will be investigated for both these conditions. Also, the investigation will be centered around parallel connected magnetic amplifier as shown in Figure #6, which is the basic circuit discussed in Chapter One.

Briefly, the information will be presented as follows: First, a group of practical design formulas based on laboratory experiments for different core materials are listed, and their use indicated. Second, the use of formulas developed in Appendix "A" for a high impedance source will be indicated. Third, the use of formulas developed in Appendix "B" for a low impedance source will be indicated. Fourth, and final will be an application of the graphs of Appendix "C" and "D" for the determination of the characteristics of s saturable core reactor with provisions to insert the rectifiers in series with the anode windings and solve graphically for the results. All the developments are for a sine waveform of supply voltage.

1. For a sine wave applied to a transformer the following equations are familiar:

$$V_s = 4.44\, N f A B_{max} \times 10^{-8} \qquad (1)$$

$$H = \frac{0.4\pi N I}{l_f} \qquad (2)$$

Since the wave shape of the voltage across a saturable core with an external load is not a sine wave the above listed formulas cannot be applied directly. Furthermore, B_{max} is difficult to measure under operating conditions. Assume that laboratory constants can be developed which are accurate enough for practical use. The following equations*, with the exception of #6 can be derived from equations #1 and #2:

$$V_s = k_v N f A \tag{3}$$

$$I_{L(max)} = \frac{k_i \, l_f}{N} \tag{4}$$

$$W_{L(max)} = 0.4 \, k_v k_i \, f A \tag{5}$$

The 0.4 was determined as a constant that gave good results for most materials.

$$T = \frac{k_t N_c^2 A}{R_c} \tag{6}$$

$$I_c = \frac{(\text{amp. turns/inch}) \times l_f}{N_c} \tag{7}$$

$$G = \frac{W_{L(max)}}{I_{cm}^2 R_c} \tag{8}$$

These equations may be rearranged as follows to solve for A, N, etc.

* From the notes of Ray E. Morgan, General Electric Co., Schenectady, N.Y.

$$A = \frac{V_s}{k_v N f} \qquad (9)$$

$$N = \frac{k_i}{I_{1(max)}} \qquad (10)$$

$$V_s = \frac{V_{L(max)}}{0.4} \qquad (11)$$

$$W_{L(max)} = 0.4\, k_v k_i f A \qquad (12)$$

$$k_v k_i = \text{core constant} \qquad (13)$$

$$A \, l_f = \text{Volume of Saturated Core Material} \qquad (14)$$

$$N_c = \frac{(T R_c)^{1/2}}{(k_t A)^{1/2}} \qquad (15)$$

Equation #6 gives the time of response, which is the time for load current to rise from a minimum value to a maximum value. This equation sets no lower limit on the time of response. However, it is well to remember that the speed of response is limited by the supply frequency, load impedance, filters, and the upper limit of resistance in the control winding, either real or reflected. The speed of response is generally increased by inserting resistance in the control winding or with the use of negative feedback. Whatever method is used to increase the response, it is done at the expense of power gain. These general statements are taken up in detail in part #4 of this chapter.

Using the equations #9 through #15 a typical design procedure would be something as follows:

A. Power Requirements

1. Impedance and power of input.
2. Impedance and power of output.
3. Required speed of response.
4. Desired performance--single ended or push-pull, etc.

B. Choice of Circuit

1. Choose the type circuit to give the required performance determined by part "A".
2. Choose the core material.
   a. Mumetal for small size high gain stages.
   b. SX-10 for 20 watts or larger with low frequency supply sources.

C. Core Size.

1. Equation #14 gives the core volume.
2. Using U punchings economical design requires that the core cross section remain within 2:1 of a square.

D. AC Coil Considerations

1. Number of turns from equation #10.
2. Supply voltage from equation #11. (Load voltage is determined in part "A".)
3. Check the area of the coil with equation #3.
4. Alter stack thickness to correct area.
5. Use 500 circular mill/amp for wire size for three legged core and 800 circular mill/amp

for spirakore or ringkore.

E. DC Coil Considerations.

1. Number of turns equation #15.
2. Choose wire size for desired signal current.
3. Signal current from equation #7.
4. Check with equation #8 to see if the power gain specifications of part "A" have been met.
5. Check step #3 in #2 to see if the size of wire is sufficient.

In order to keep the amount of material used at a minimum, equation #12 indicates that a large $k_v k_i$ is desirable. For small sizes the material with the large $k_v k_i$ is usually chosen. However, for large power outputs it is possible that a poorer material may be somewhat larger but less expensive.

2. If the impedance of the control winding is high then there will be little or no circulating current in the control winding due to transformer action of the anode windings. The transformer action referred to is present only as long as the core is not saturated.

Assuming the core would magnetize and demagnetize along the upper magnetization curve of the hysteresis loop, the time in the supply frequency cycle when the core saturates can be represented by the following equation.

$$\theta_f = \cos^{-1}\left(\frac{B_m - B_f + B_o}{B_m}\right) \qquad (16)$$

Equation #15 is shown in graph form on graph #1 A1 of Appendix "A". B_f is dependent upon the core and is determined in the laboratory. B_m is dependent upon the supply voltage and is determined by:

$$B_m = \frac{E_m \times 10^{-8}}{\omega NA} \qquad (17)$$

E_m is the maximum value of the supply voltage. N is the number of turns on the anode winding and A is the core cross section area. B_o is the flux density of the core when no current is flowing in the anode winding. B_o is determined with the aid of the magnetization curve and represents the point "A" in Figure #3. Point "A" shifts as the current in the control winding is changed. For a fixed supply voltage B_m/B_f is a constant and is shown as a solid line on the graph. A sample is taken to illustrate the use of graph #A1. Assume B_m/B_f equal to one and the ratio of the initial flux density to be zero. Then the reactor would saturate at approximately 90 degrees. Knowing this the average or rms of the current can be computed for a specific value of load resistance. Letting the ratio of B_o/B_f vary would give the varying angle of firing from which the varying load current could be computed. This information is usually graphed in the form of Figure #5.

Similar information can also be obtained from graph #2A. $e_L\Big|_{\theta_f}^{\pi}$ is the average voltage across the load produced by one reactor. The total load voltage is twice this value. Graph #2A is used as follows: Choose a supply voltage,

compute the ratio of B_m/B_f which will give the correct line to follow along. Next, choose a value of the control current and compute B_o. Take the ratio of B_o/B_m and enter the graph to the B_m/B_f line then across to the output. This value of output must next be multiplied by E_f and divided by 2π to obtain the average load voltage. If the average output voltage is known then it is easy to compute the current for varying values of load resistance.

3. If the impedance of the control winding circuit is low then it is reasonable to expect currents to circulate in the control windings due to transformer coupling from the anode windings. This circulating of currents will influence the firing of the reactor and it is reasonable to expect the operation to be somewhat different than the case where the high impedance in the control circuit prevents the circulation of induced currents.

The three important periods of operation are, the conduction of current by anode #1, the conduction of current by anode #2, and the simultaneous conduction of current by both anode windings. The equations for the currents are developed in detail in Appendix B for each condition. Let i_1 be the current in the control winding and i_2 and i_3 be the currents in the two anode windings.

For the period that the current i_3 is zero

$$i_1 = \frac{\ell}{N}\left[(H_0 + H_2\cos 2\omega t \ldots) - (H_1 \sin\omega t + H_3 \sin 3\omega t \ldots)\right] \quad (18)$$

$$i_2 = \frac{2l}{N_2}\left[H_1 \sin \omega t + H_3 \sin 3\omega t \ldots\right] \qquad (19)$$

$$i_3 = 0$$

For the period that the current i_2 is zero

$$i_1 = \frac{l}{N_1}\left[(H_0 + H_2 \cos \omega t \ldots) + (H_1 \sin \omega t + H_3 \sin 3\omega t \ldots)\right] \qquad (20)$$

$$i_2 = 0$$

$$i_3 = \frac{2l}{N_2}\left[H_1 \sin \omega t + H_3 \sin 3\omega t \ldots\right] \qquad (21)$$

For the period that i_2 and i_3 are conducting simultaneously

$$i_1 = \frac{1}{1+k'}\left[\frac{k'l}{N_1}(H_0 + H_2 \cos 2\omega t \ldots) + N_1 I_1\right] \qquad (22)$$

$$i_2 = \frac{l}{N_2}\left[H_1 \sin \omega t + H_3 \sin 3\omega t \ldots\right] + \frac{1}{(1+k')N_2}\left[l(H_0 + H_2 \cos 2\omega t \ldots) - N_1 I_1\right] \qquad (23)$$

$$i_3 = \frac{l}{N_2}\left[(H_1 \sin \omega t + H_3 \sin 3\omega t \ldots)\right] - \frac{1}{(1+k')N_2}\left[l(H_0 + H_2 \cos 2\omega t \ldots) - N_1 I_1\right] \qquad (24)$$

The coefficient of the harmonics are Bessel functions of the first kind and the Nth order and are computed with the aid of the table at the end of Appendix "B".

To determine the characteristics of an amplifier the following procedure may be used:

a. Adjust the constants of the analytical expression $H = U\sinh(uB) + vB$ to fit the magnetization curve of the material.

b. Calculate B_m using equation #17.

c. Assume a value of B_o.

d. Determine σ the time at which the current I_3 in the second anode begins to flow.

e. Calculate $N_1 I_1$ with the following equation

$$N_1 I_1 = \ell\left[(H_0 + H_2 \cos\sigma \ldots) - (1+k')(H_1 \sin\sigma + H_3 \sin 3\sigma t \ldots)\right] \qquad (25)$$

This permits computing the I_1 for the assumed B_0 in "c" above.

f. Calculate the currents for the three periods of conduction*. Knowing the current expressed in the series it is possible to compute the average or rms value of the current.

If additional external feedback is to be used then the average value of the current becomes important. Since i_2 and i_3 are similar in their respective modes the average value during one half cycle is the true average value.

$$(i_2 + i_3)_{ave} = \frac{4\ell}{\pi N_2}\left[H_1 + \frac{H_3}{3} + \frac{H_5}{5} \ldots\right] \qquad (26)$$

Generally

$$(i_2 + i_3) = \frac{2\ell}{N_2}\left[H_1 \sin\omega t + H_3 \sin 3\omega t \ldots\right] \qquad (27)$$

It might be well to mention that this average value is good for feedback computation only if it is filtered, and then it is valid only in a steady state condition.

4. Formulas have now been developed for the self saturated magnetic amplifier using laboratory constants, high impedance control source, and low impedance control

* Analytical Determination of Characteristics of Magnetic Amplifiers with Feedback. D. W. VerPlank; L. A. Finzi; D. C. Beaumariage. AIEE Tech. Paper 49-139.

sources which assist in predictions for design purposes. There is another way of solving the amplifier problem and this time the saturated reactor is first investigated, then the self saturated feature due to the insertion of rectifiers is treated as positive feedback. To do this a set of universal curves* are first prepared and from these curves the performance of the saturable reactor can be predicted.

The assumptions and approximations are listed under Appendix C. The circuit under consideration is Figure #6, without the rectifiers. After this circuit is solved then the rectifiers are added and the feedback included.

The dc magnetic curve is approximated analytically by

$$i_2 = CU + C_n U^n \tag{28}$$

C, C_n, and n are constants depending upon the shape of the magnetization curve. The current of one anode coil is i_2, and U the flux linkage of the anode coil. The sharper the bend at the knee of the dc magnetization curve, the higher the power of N required to achieve a fit. This is shown on a per unit basis for the lines labelled $I_{opu-15} = 0$ in graph #1 being sharper than $I_{opu-5} = 0$.

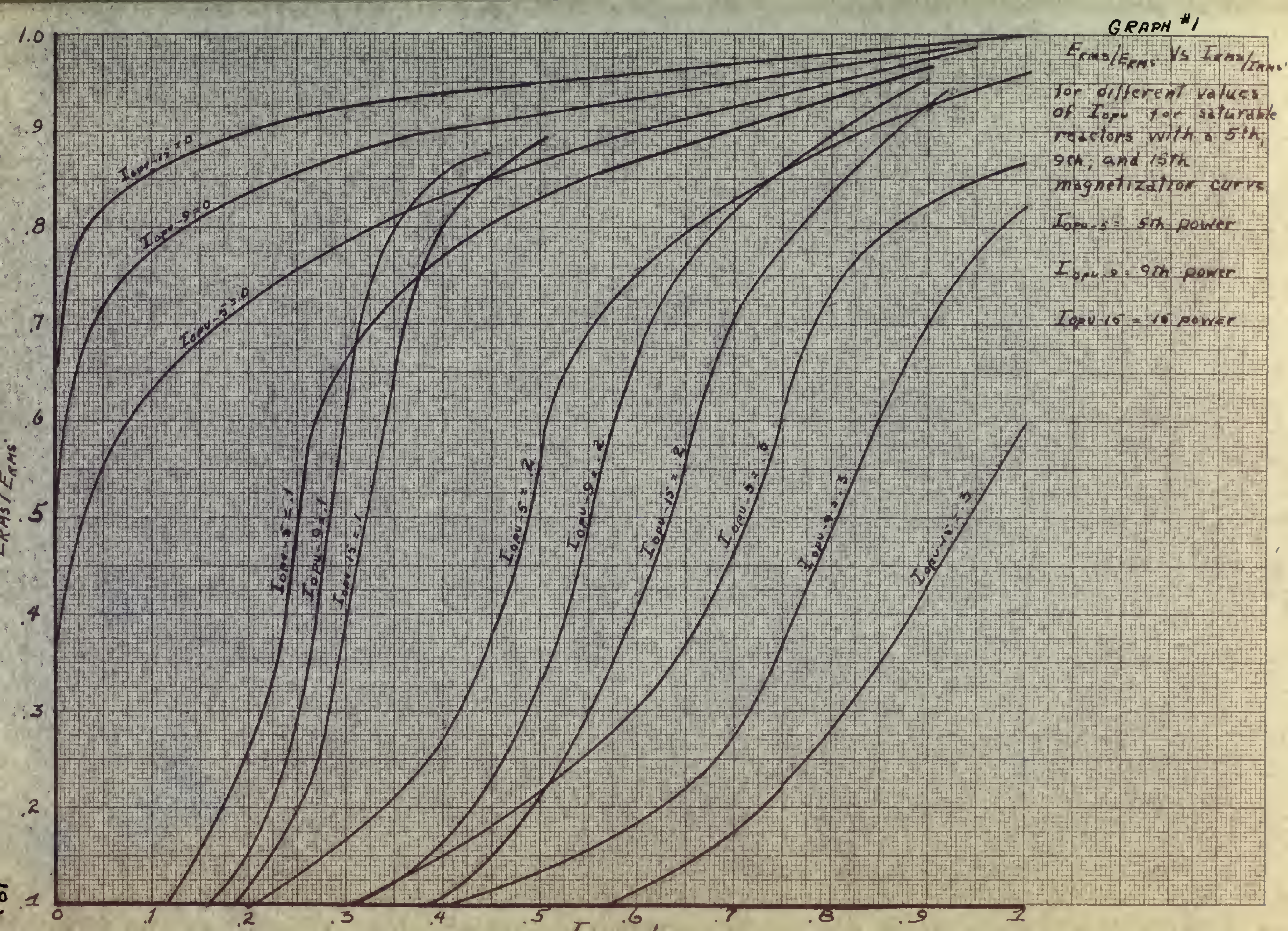
Graphs #1C and #2C show the analytical results less the straight line portion of the magnetic curve on a per unit basis for the rms and average basis. If it is found

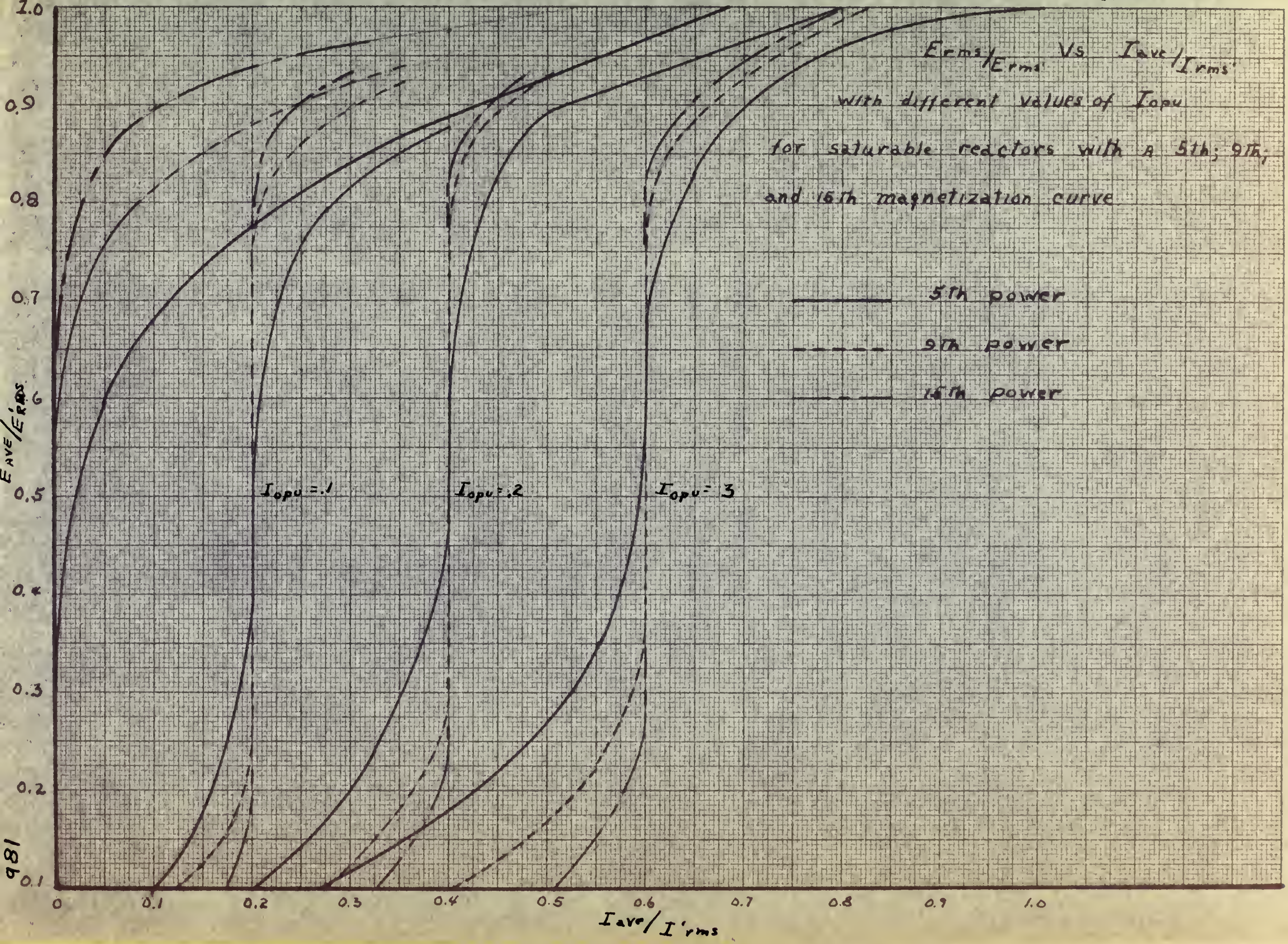
* Universal Curves for D. C. Saturated Reactors: Boyd C. Merrill, June 1948, Princeton University.

that other values of N are required then interpolation can be used on the graphs.

Graphs #1 and #2 as stated are the per unit relation between the current and voltage less the straight line portion of the power series to represent different magnetic cores. The I_{opu} values are inserted representing the current in the control winding on a per unit basis.

When I_{opu} is zero, note that a higher power function gives a sharper bending curve. This is a representation of the knee of the saturation curve. The per unit load current of one material compared to another is higher as the magnetization curve is represented by the higher power function. For a given value of control current in similar amplifiers one would then expect to have a larger load current from the core with the sharper bend at the knee of the magnetization curve. Also, note that the per unit change in control current decreases with higher power curves for any specified change in power output current. This indicates the desirability of a core having a steep magnetization curve with a sharp break to increase the gain, and such has been the case experimentally. However, the reader should remember at this point that the assumption has been made that once a material saturates the coil incremental inductance is the air inductance of the coil and usually neglected. This may not be the case with some materials, especially those with magnetization curves having steep initial slopes and sharp knees. The incremental inductance may remain greater than air and cannot be neglected.


GRAPH #1
Erms/ERMS' Vs Irms/IRMS'
for different values of Iopu for saturable reactors with a 5th, 9th, and 15th magnetization curve
Iopu-5 = 5th power
Iopu-9 = 9th power
Iopu-15 = 15 power
Iopu-15 = 0
Iopu-9 = 0
Iopu-5 = 0
Iopu-5 = .1
Iopu-9 = .1
Iopu-15 = .1
Iopu-5 = .2
Iopu-9 = .2
Iopu-15 = .2
Iopu-5 = .3
Iopu-9 = .3
Iopu-15 = .3
Erms/ERMS'
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1


$E_{rms}/E_{rms'}$ Vs $I_{ave}/I_{rms'}$
with different values of I_{opu}
for saturable reactors with a 5th; 9th;
and 15th magnetization curve
5th power
9th power
15th power
$I_{opu}=.1$
$I_{opu}=.2$
$I_{opu}=.3$
E_{ave}/E'_{rms}
I_{ave}/I'_{rms}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

An example is the design of large power amplifiers where the curves may lead to a solution that is not the best from over-all considerations. A material that saturates more slowly, but the magnetization curve of which has a flat top is the better for this application. This information is included to remind the reader that there are limitations to all these developments, which must be kept in mind for any particular problem.

Since there are several lines on graph #1, take for example: The voltage vs. the output current of a reactor connected as in Figure #6, was graphed for various values of current in the control winding, and shown on graph #3. The maximum voltage of interest is 200 volts and is labelled E'_{rms}. The current, I_{rms} is 28.9 amps for E'_{rms} with zero control current conditions. The curve was next reduced to a per unit basis but not shown, and found to agree with the ninth power curve. The ninth power curve is more clearly shown in graph #1C.

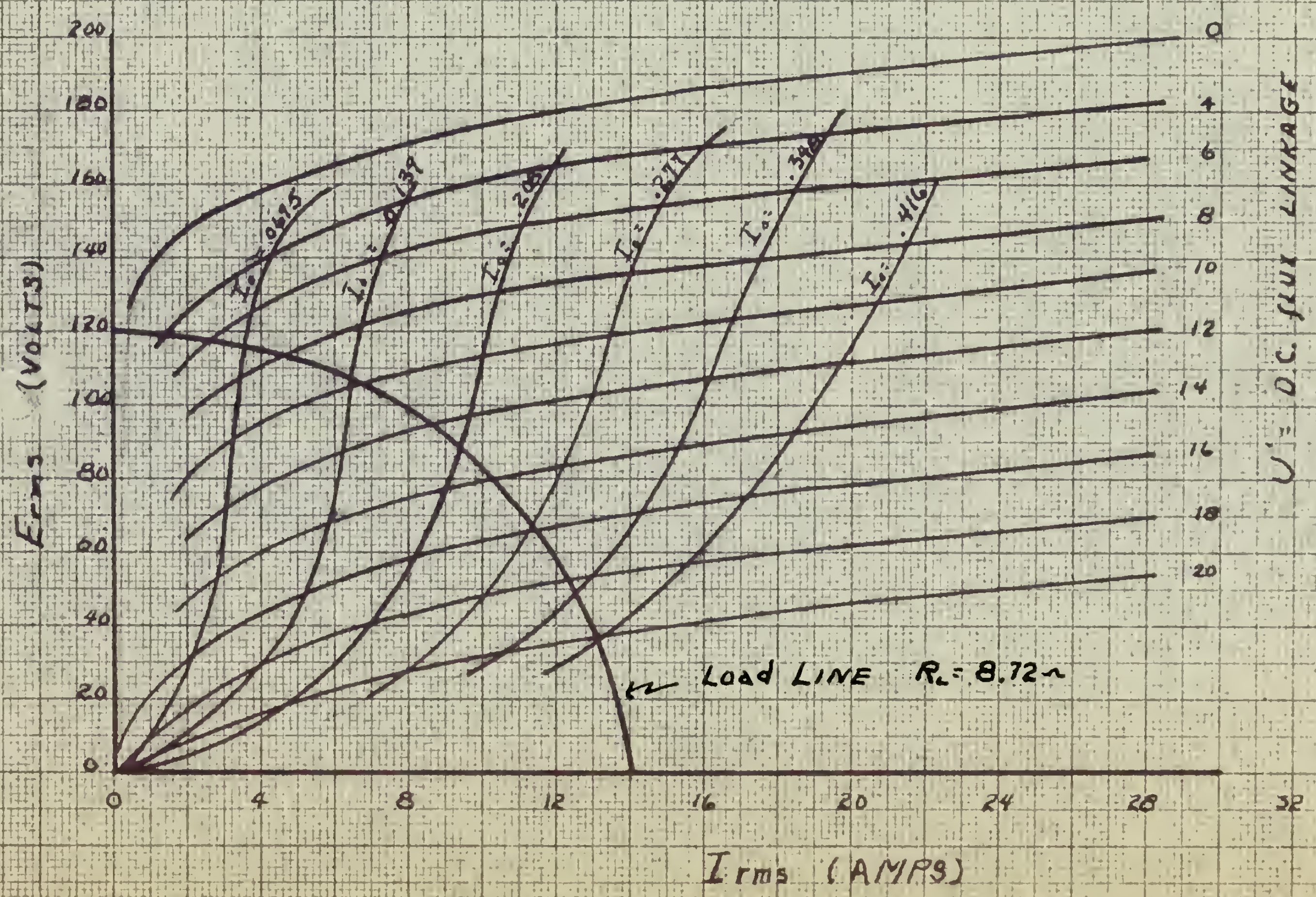
The next step is to superimpose the dc flux linkage of the control winding upon graph #3. To do this, let U_o' be the dc flux linkage; then, the dc flux is $\frac{U_o'}{N_{dc}}$

In a like manner let U_o be the ac flux linkage of one anode coil due to current in the control winding. The anode dc flux is $\frac{2U_o}{N_{ac}}$. Since there are two anode coils the total anode flux is

$$\frac{U_o'}{N_{dc}} = \frac{2U_o}{N_{ac}} \tag{29}$$

GRAPH #3

GE 666912 REACTOR CHARACTERISTIC CHART



Giving $U_o' = 2\,U_o \dfrac{N_{dc}}{N_{ac}}$ (29a)

$$N_{dc} = 230, \quad N_{dc} = 4{,}000, \quad r = \frac{U_o}{U_m}$$

$$U_o' = 34.75\,U_o$$

$$E_{rms}' = 200 \text{ volts}$$

If $U_o' = 10$ then $U_o = 0.2875$

Next determine a single point on the U_o' 10 line as follows: For E_{rms} = 140 volts, E_{rpu} = 0.7. U_m represents the peak value of the total flux. Therefore, for a 60 cycle supply as used in graph #3

$$r = \frac{U_o}{U_m} = \frac{0.2875\,(377)}{140\sqrt{2}} = 0.548 \qquad (30)$$

Looking at graph #1C we see I_{rpu} 1.0 for E_{rpu} 0.7; r 0.548. This indicates the maximum output or 28.9 amps. Other points are obtained by choosing other values of anode voltage. This line U_o' 10 is drawn on graph #3 by connecting the computed points. Similarly other lines for different values of U_o' can be computed and drawn on the reactor characteristic chart. Experience shows that even though the inductance of the reactor is not a constant an elliptical load line is a good approximation, and is accurate enough. The equation of the load line is:

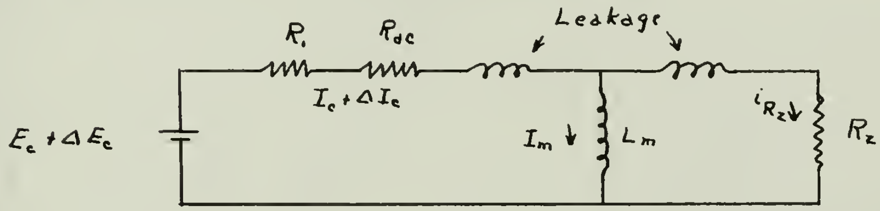
$$\frac{(\text{reactor voltage})^2}{(\text{line voltage})^2} + \frac{(\text{line current x } R_L)^2}{(\text{line voltage})^2} = 1. \qquad (31)$$

A load line for a load resistance of 8.72 ohms is

also inserted on graph #3. Having constructed the load line it is convenient to next draw a graph of I_{rms} vs. $I_{cont.}$. This is done on graph #4 and compared to experimental results for a supply voltage of 120 volts. Other graphs could be constructed for different voltages thus giving a family of curves.

For further investigation of the reactor, U_o' vs. I_m is obtained by plotting this information from graph #3 (I_m is equal to $I_{control}$ in the steady state condition). The slope of this curve is the incremental inductance.

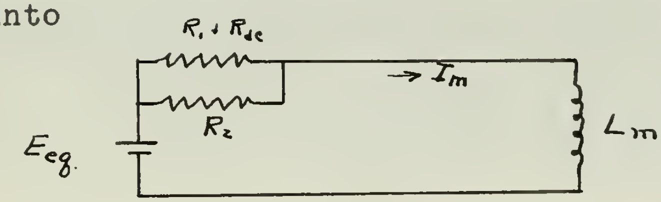
With the ac coils connected in parallel as in Figure #6 the equivalent circuit shown schematically represents a change in the control circuit voltage.

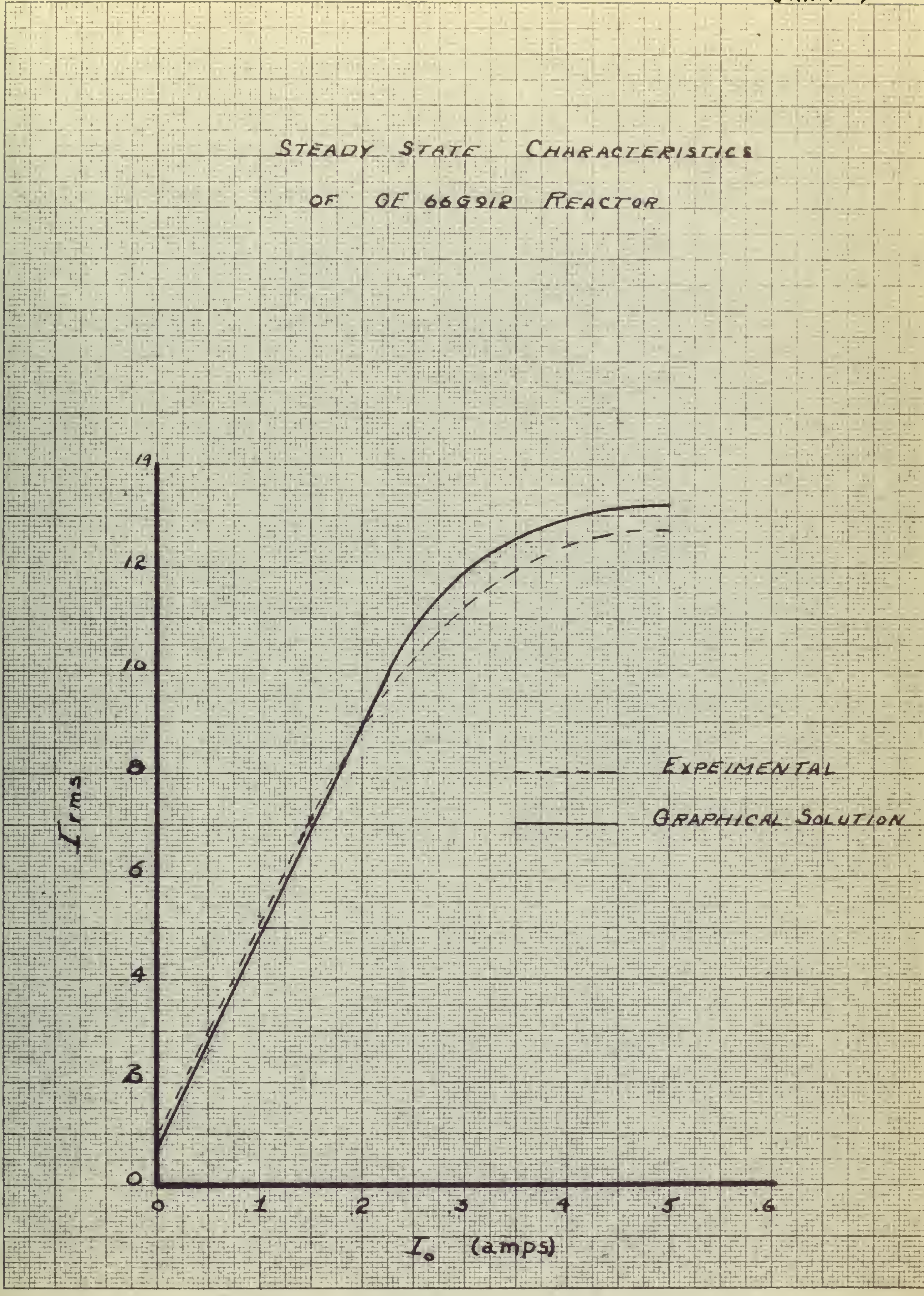


$$R_z = R_{ac}\frac{(N_{dc})^2}{(N_{ac})^2}$$

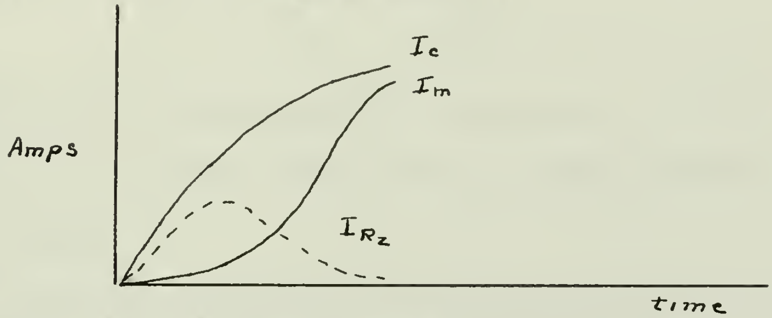
i_{R_z} represents the equivalent of the induced current in the anode windings.

R_z is the resistance of the anode windings reflected into the control winding. The leakage reactance is small and is usually neglected. Any change in $I_{control}$ does not immediately produce a change in I_m but changes i. Neglecting leakage reactance the circuit can be simplified by Thevenin's theorem into




STEADY STATE CHARACTERISTICS
OF GE 66G912 REACTOR
Irms
0
2
4
6
8
10
12
14
0
.1
.2
.3
.4
.5
.6
I₀ (amps)
EXPEIMENTAL
GRAPHICAL SOLUTION

Investigating the equivalent diagram it is seen that R_z is the maximum value of the loop resistance. For good anode windings design their resistance should be as low as practicable. This is one limit to the response that a saturated reactor can have. The L_m is not a constant as seen by the slope of graph #4. Pictorially the current can be represented by the following sketch:



A method for finding the I_m analytically is not known. Appendix D presents one graphical solution for the determination of the response of the amplifier which is an indication of I_m. For the GE 66G912 Reactor Characteristics the control circuit used was a 230 volt supply.

The steady state current would be 0.4 amps. However, suppose the time required for the magnetization current to increase from zero to 0.126 amps was desired. The steps are laid off in graph #5 in accordance with the method developed in Appendix D.

$$\Sigma h = 10.65$$

$$\Delta t = \frac{h}{kR} = \frac{10.65}{0.4(195)} = 0.1365 \text{ sec.}$$

This method of solution gave an accuracy of 15%. Similarly a decrease in control current can be approximated with the aid of a graph.

The graphical methods of determining the characteristics of the reactor are given to show the methods of application to a reactor in general. Next, to obtain the characteristics of an amplifier which has rectifiers in series with the anode winding, a feedback factor is used.

If feedback is either positive or negative it is convenient to define a feedback factor similar to that for other type circuits.

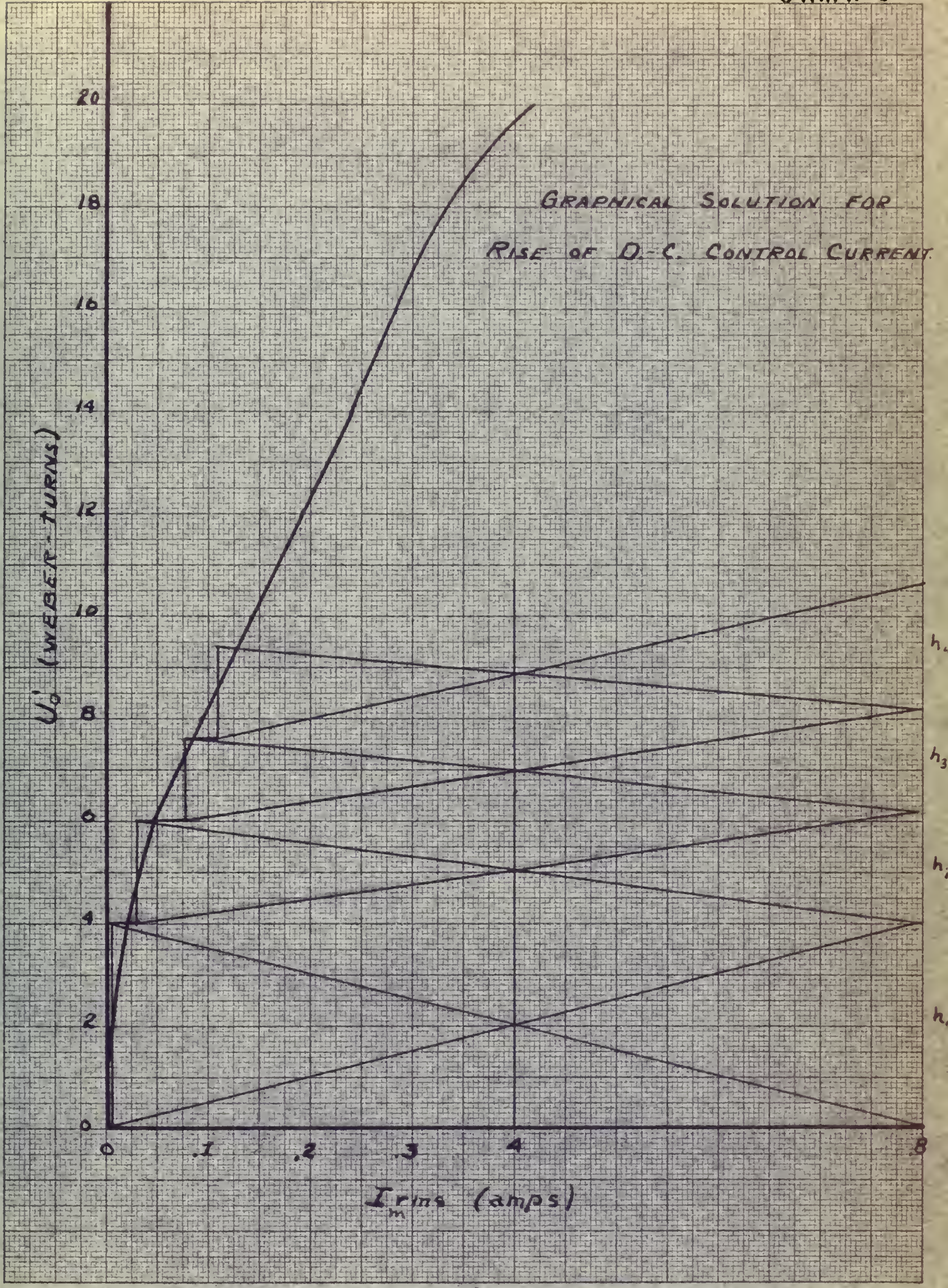
$$\beta = \frac{\text{(Fed back d.c.)}}{\text{(Total ave. load current)}} \times \frac{\text{(Feedback turns)}}{N_{ac}} \qquad (32)$$

If the feedback is internal the N_{ac} is the turns on one anode winding. With feedback the control mmf is made up of two parts;--one from the control circuit and the other the feedback circuit

$$N_{dc} I_m = I_c N_{dc} + \beta I_{ave} N_{ac} \qquad (33)$$

$$I_c = I_m - \beta I_{ave} \frac{N_{ac}}{N_{dc}} \qquad (34)$$

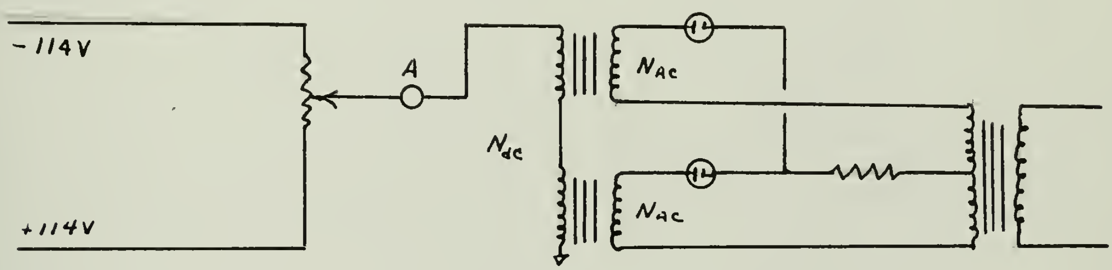
GRAPH 5


GRAPHICAL SOLUTION FOR
RISE OF D.-C. CONTROL CURRENT
U'o (WEBER-TURNS)
Irms (amps)
20
18
16
14
12
10
8
6
4
2
0
.1
.2
.3

For self feedback the assumption is made that the B vs. NI curve passes through zero. Substituting in the equation for β gives a feedback factor of ½.

$$I_c = I_m - \frac{I_{ave}}{2} \frac{N_{ac}}{N_{dc}} \tag{35}$$

Since feedback is based on average load current, a curve of the GE 66G912 reactor characteristics are drawn on an average current basis. The resulting curves are shown in graph #6 based on graph #2C. Graph #6 is used for the self saturating circuit shown.



The U_O' are placed on graph #6 in a manner similar to those illustrated for graph #3.

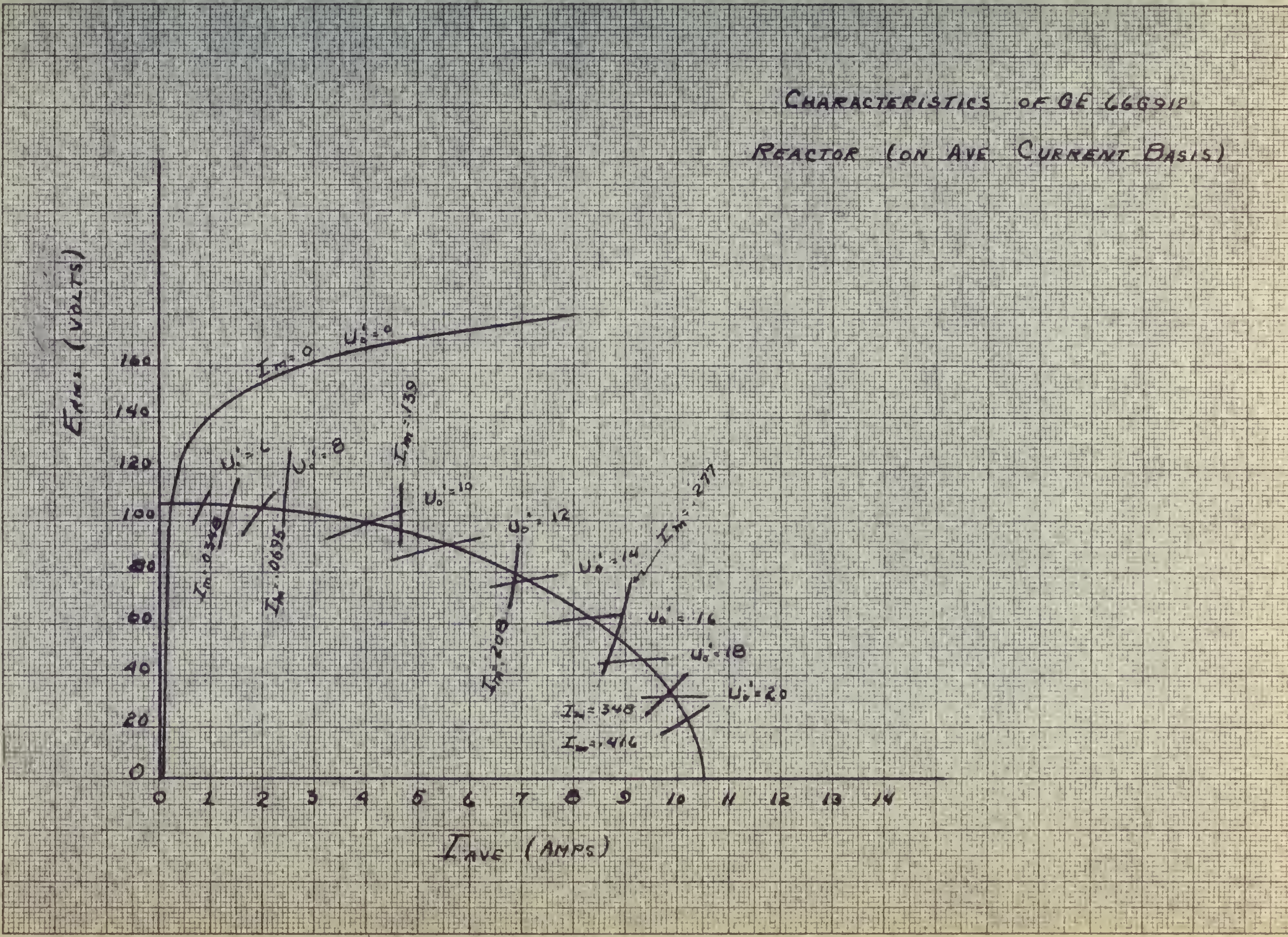
A sample calculation of the control current required using equation #35 and graph #6 is shown next.

For $I_m = 0.227 \qquad I_{ave} = 8.79$ Amps

$N_{ac} = 230 \qquad N_{dc} = 4{,}000$

$$I_c = 0.277 - \frac{1}{2}(8.79)\frac{230}{4000} = 25 \text{ ma.}$$

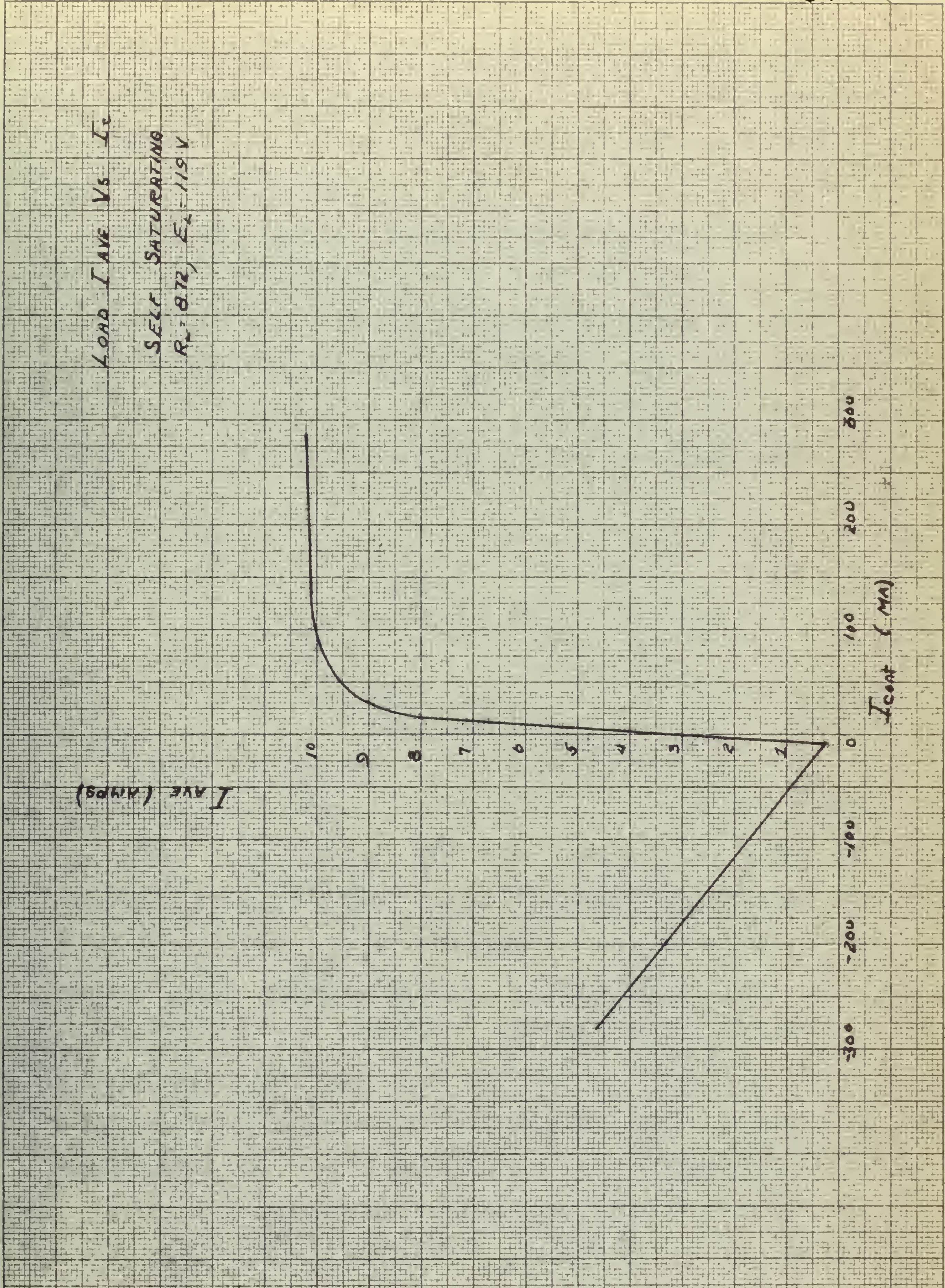
A graph of I_c vs. $I_{ave.}$ is shown on graph #7. No experimental curve is available to compare with the computed results, however, they are easy to obtain. With positive feedback the hysteresis loop is relatively larger. It was



CHARACTERISTICS OF GE 666912
REACTOR (ON AVE CURRENT BASIS)
E RMS (VOLTS)
0
20
40
60
80
100
120
140
160
I AVE (AMPS)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
Im = 0
U₀' = 0
Im = .0348
U₀' = 6
Im = .0695
U₀' = 8
Im = .139
U₀' = 10
U₀' = 12
Im = .208
U₀' = 14
Im = .277
U₀' = 16
U₀' = 18
U₀' = 20
Im = .348
Im = .416

GRAPH 6

GRAPH 7


LOAD I AVE Vs I_c
SELF SATURATING
R_L = 8.72, E_L = 119 V
I AVE (AMPS)
10
9
8
7
6
5
4
3
2
1
-300
-200
-100
0
100
200
300
I cont (MA)

present without feedback but was masked by the large value of control current required. Repeating again, this feedback is present due to the rectifier preventing the flux from reversing in the core.

Since the control current required with rectifiers in the anode coils in the circuit is less, then the gain of the amplifier has been increased. The smaller change of control current for a given change of U_o' results in a larger effective inductance. If the control circuit remains the same, then for small changes of the control current, the following is approximate:

$$L = \frac{\Delta U_o'}{\Delta I_c} \tag{37}$$

$$t_c = \frac{L}{R_c} = \frac{\Delta U_o'}{(\Delta I_c) R_c} \tag{38}$$

A given change in load current, for any one value of load resistance, requires a given change in the dc flux regardless of how this change is obtained. Hence, if the effective control circuit resistance remains a constant and feedback reduces the required change of control current the response time of the circuit has been increased. Now, if the control circuit resistance were increased until the response time were where it was without the added rectifiers, the gain has decreased to about its value before feedback was introduced. The response and gain of an amplifier are interdependent.

To analyse the behavior of the reactor with feedback during transits it is convenient to graph U_o' vs. I_c. To do

this, values of I_c are computed from the values of I_m using graph #6 and these computed values are plotted against the interpolated value of U_o'. This result is shown on graph #8. Since an important region of operation is around the steepest slope, the transits were taken around I_c equal to zero.

For small changes of current the slope of U_o' vs. I_c changes slowly around the origin. For an approximate solution L in this region can be assumed a constant. This approximation leads to an exponential transit. Sample computation of the transit rise with feedback is done with the aid of graph #8. If the control current increases from zero to 9 ma. the E_{eq} was found to be 1.25 volts and R_{eq} equal to 139 ohms.

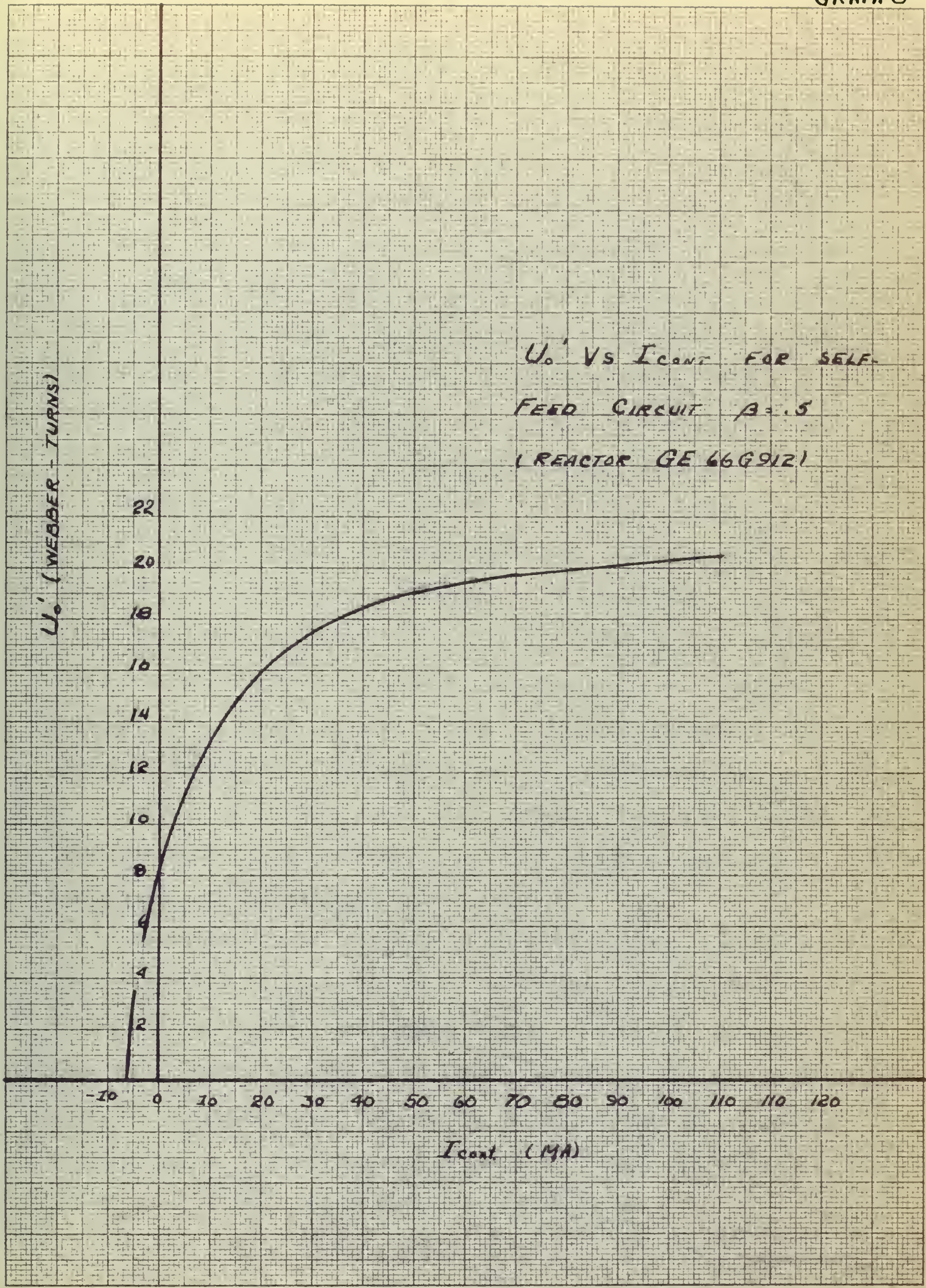
$$L = \frac{\Delta U_o'}{\Delta I_c} = \frac{12.9 - 8.2}{9 \times 10^{-3}} = 522 \text{ henries} \tag{39}$$

From this value of L the conventional time constant is

$$t_c = \frac{L}{R_c} = \frac{522}{139} = 3.75 \text{ seconds} \tag{40}$$

The time constant was determined to be 3.3 seconds with the aid of an oscillogram. The difference is not surprising considering the number of approximations and simplifications that were necessary to arrive at the solution.

Negative feedback reduces the time of response and the gain of the amplifier. This type of feedback must be inserted externally.



U_o' Vs I_{cont} FOR SELF-FEED CIRCUIT $\beta = .5$
(REACTOR GE 66G912)
U_o' (WEBBER - TURNS)
22
20
18
16
14
12
10
8
6
4
2
-10
0
10
20
30
40
50
60
70
80
90
100
110
110
120
I_{cont} (MA)

SUMMARY

All of the methods presented require considerable preliminary preparation before the operation of the magnetization amplifier can be determined. It should also be pointed out here that the solutions are applicable only to resistance loads. The practical design formulas #2 through #8 are easy to use once the laboratory constants are determined and work quite well for low frequencies of supply voltage. The one item of interest not included which would be very useful is the minimum load current for a given supply voltage.

The computing of the angle of the supply frequency at which the core saturates is not difficult with the aid of graph #1A. The current is assumed to be resistance limited after core saturation and from this the wave form of the current can be constructed. Within the limits of the simplifications this method is well suited to the explaining of the cycle of operation. The use of the upper magnetization curve is a step in the proper direction; however, it is well known that the path of magnetization is a curve lying below the upper loop of the demagnetization curve.

The computing of the load current with a low impedance control source is more involved due to the effect of the induced current in the control winding. In general, this low impedance loop of the control source increases the response time of the amplifier. Its action is similar to that of a copper slug on a slow release relay. It delays a sudden change in the mean value of the magnetic flux.

The use of Universal Curves to determine the operation for different type core materials is as easy as the other methods. This method shows the increased sensitivity resulting from the introduction of the rectifiers in series with the anode windings. A feedback factor of one half is an approximation based on the use of the dc magnetization curve. This factor would vary for different core materials, since the magnetization and demagnetization curves do not pass through the origin.

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9. Universal Curves for D.C. Saturated Reactors, Boyd C. Merrill, June 1948, Princeton University.

APPENDIX A

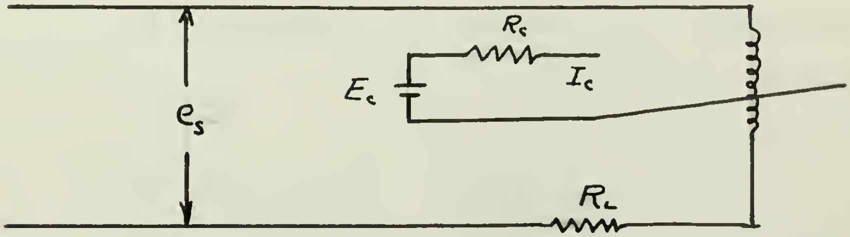
Determination of the Time of Core Saturation.

Assumptions and Approximation

1. The forward resistance of the rectifier is zero.
2. The back resistance of the rectifier is infinite.
3. The reactor core saturates and de-saturates along the upper magnetization curve of the hysteresis loop. This assumption is good for materials having small hysteresis loops since the rectifiers prevent the current from reversing in the anode coils.

APPENDIX A

If one core of the parallel connected self saturating magnetic amplifier is considered the problem is simplified. For the following circuit the differential equation is



$$e_s = N\frac{d\varphi}{dt}10^{-8} + i_2 R_L + e_r \qquad (1)$$

For a sine wave of applied voltage equation #1 becomes

$$E_m \sin\omega t = N\frac{d\varphi}{dt}\times 10^{-8} + i_2 R_L + e_r \qquad (2)$$

If the rectifier voltage is small with respect to E_m equation #2 can be approximated as follows:

$$E_m \sin\omega t = N\frac{d\varphi}{dt} + i_2 R_L \qquad (3)$$

$$N\frac{d\varphi}{dt} = E_m \sin\omega t - i_2 R_L \qquad (4)$$

$$B = f(H + H_o) \qquad (5)$$

H_o is the magnetizing force contributed by the current in the control winding and H is the magnetizing force contributed by current in the anode winding. In general

$$H = \frac{0.4\pi NI}{l_f} \qquad (6)$$

Since the rectifier in series with the anode winding prevents the reversal of current the direction of H does not reverse in the anode winding. In general

$$d\varphi = A\,dB \tag{7}$$

Substituting from equation #7 and #6 into equation #4 gives

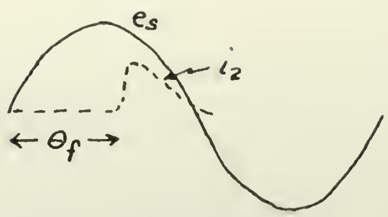
$$dB = \left(\frac{E_m \sin \omega t}{NA} - \frac{H l R_L}{4\pi N^2 A}\right) dt \times 10^{-8} \tag{8}$$

Substitute for dt in equation #8. is the incremental change in radians of the supply frequency for an incremental change in B.

$$dB = \left(E_m \sin \omega t - \frac{H l R_L}{4\pi N}\right) \frac{d\theta}{\omega N A} \times 10^{-8} \tag{9}$$

With the aid of the magnetization curve a change in the supply frequency can be solved for in terms of E_m and H for a chosen dB. H is chosen from the B vs. H curve for the location of dB. This H read from the B vs. H curve must then be corrected for the control current's H_0 before the instantaneous anode current can be computed. The wave form of the load voltage iR_1 can be plotted for $\theta + \Delta\theta$ This plot would apply for a given value of control current. The above sample includes the anode coil resistance in the load resistance. This can be corrected for if the anode resistance is not small with respect to the load resistance.

The resulting curve would have the following form which would include the small values of current that are present prior to saturation of the core.



Since the core does not saturate along the upper saturation curve the current before firing will be slightly larger in value. However, this portion of the current is usually small and will be neglected as listed in the approximations below.

By making the approximations that the pre-saturation current is zero, and that the current in the control winding will not saturate the core in the negative portion of the B-H curve the solution is simplified.* This restriction eliminates the mode $-B_f \doteq B_o \doteq B_f$

Since the current prior to the core's saturation has been neglected equation #8 simplifies to the following, for the pre-saturation period.

$$dB = \frac{E_m \sin \omega t}{NA} \, dt \times 10^{-8} \tag{10}$$

Firing or saturation occurs when the critical value B_f at the knee of the B-H curve has been reached.

$$B_f = B_o + 10^{-8} \int_0^{t_f} \frac{E_m}{NA} \sin \omega t \, dt \tag{11}$$

$$B_f - B_o = \frac{E_m 10^{-8}}{\omega NA} (1 - \cos \omega t_f) \tag{12}$$

$$\theta_f = \omega t_f$$

$$\theta_f = \cos^{-1} \left[1 - \frac{(B_f - B_o)\,\omega NA}{E_m 10^{-8}} \right] \tag{13}$$

*Self Saturating Magnetic Amplifiers, AIEE Tech Paper 49-140 by W. J. Dornhoefer.

$$B_m = \frac{E_m \, 10^{-8}}{\omega N A} \tag{14}$$

$$\theta_f = \cos^{-1} \left(\frac{B_m - B_f + B_o}{B_m} \right) \tag{15}$$

Because the rectifiers prevent the reactor core from saturating on alternate half cycles the value of can be restricted to $0 \leq \theta_f \geq \pi$ Equation #15 can be plotted by showing θ_f as a function of B_o/B_f, with B_m/B_f as a parameter. For example let $B_m/B_f = 1$.

$$\theta_f = \cos^{-1} \frac{B_f}{B_m} \left(\frac{B_m}{B_f} + \frac{B_o}{B_f} - 1 \right) = \cos^{-1} \frac{B_o}{B_m} \tag{16}$$

$$\theta_f = \cos^{-1} \frac{B_o}{B_f}$$

SAMPLE PLOT

B_m/B_f	B_o/B_f	f
1	-1.0	180.0
	-0.8	143.1
	-0.6	126.9
	-0.4	113.6
	-0.2	101.5
	-0.0	90.0
	0.2	78.5
	0.4	66.4
	0.6	53.1
	0.8	36.9
	1.0	0.0

Computation of average voltage across the load neglecting the prefiring skirt and assuming all the resistance is in the load, then for one reactor

$$e_L\Big|_{\theta_f}^{\pi} = \frac{1}{2\pi}\int_{\theta_f}^{\pi} E_m \sin\theta \, d\theta = \frac{E_m}{2\pi}\left(1+\cos\theta_f\right) \tag{17}$$

Substituting for θ_f

$$e_L\Big|_{\theta_f}^{\pi} = \frac{E_m}{2\pi}\left[\frac{B_m + B_m + B_o - B_f}{B_m}\right] \tag{18}$$

$$e_L\Big|_{\theta_f}^{\pi} = \frac{\omega N A B_m 10^{8}}{2\pi}\left[2B_m + B_o - B_f\right] \tag{19}$$

Defining $\qquad E_f = \omega N A B_f \, 10^{-8}$ (19a)

$$\frac{e_L\Big|_{\theta_f}^{\pi}}{\frac{E_f}{2\pi}} = \frac{2B_m}{B_f} + \frac{B_o}{B_f} - 1 \; ; \quad 0 \leq \theta \leq 180 \tag{20}$$

$$0 \leq \frac{E_L\Big|_{\theta_f}^{\pi}}{\frac{E_f}{2\pi}} \leq \frac{2E_m}{E_f} \tag{21}$$

The maximum value of $e_L\Big|_{\theta_f}^{\pi}$ occurs as seen by equation (17) for θ equal to zero. This value is obtained when the core is saturated by the control winding.

The idealized post firing average output for a half wave self-saturating magnetic amplifier may now be plotted as a function of $\frac{B_o}{B_f}$ with $\frac{B_m}{B_f}$ as a parameter. The full wave magnetic amplifier should have twice this value under the same simplifying assumptions.

For plotting let

$$\text{Output} = \frac{e_L\Big|_{\theta_f}^{\pi}}{\frac{E_f}{2\pi}}$$

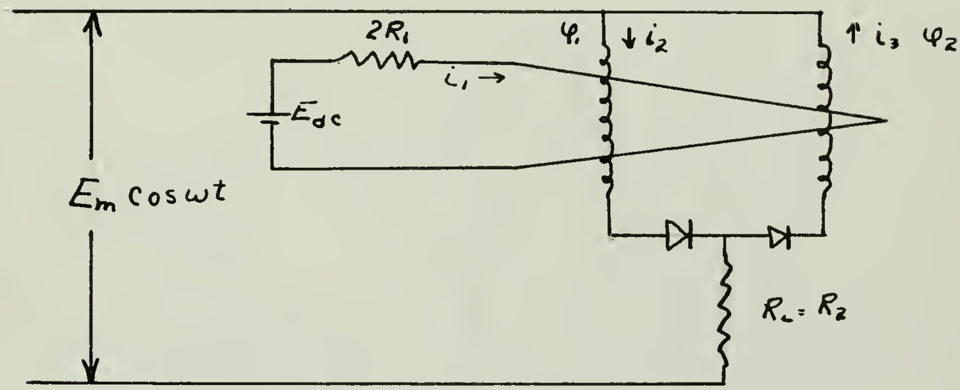
To convert the graph for the average output voltage

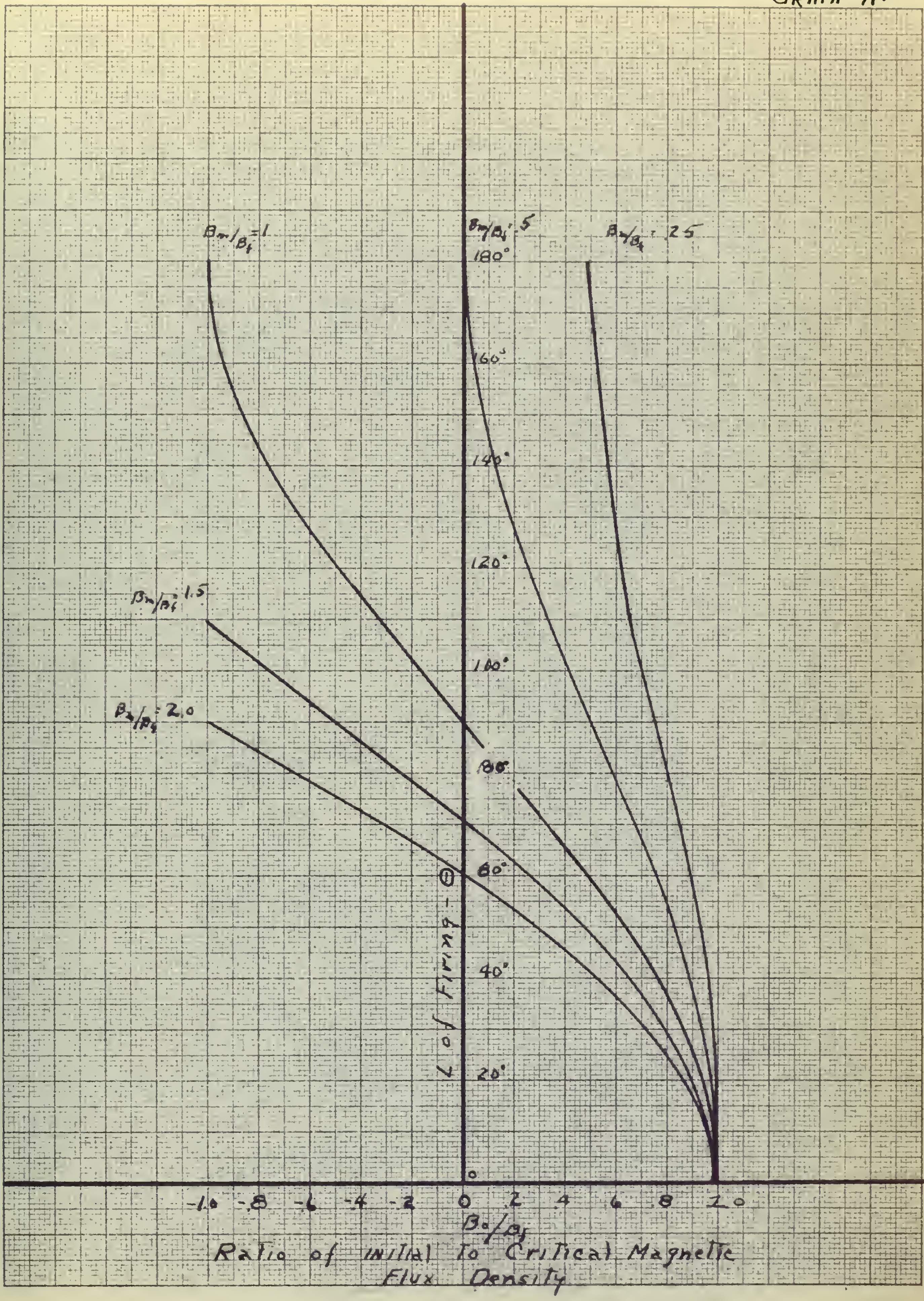
simply multiply the readings by $\frac{E_f}{2\pi}$

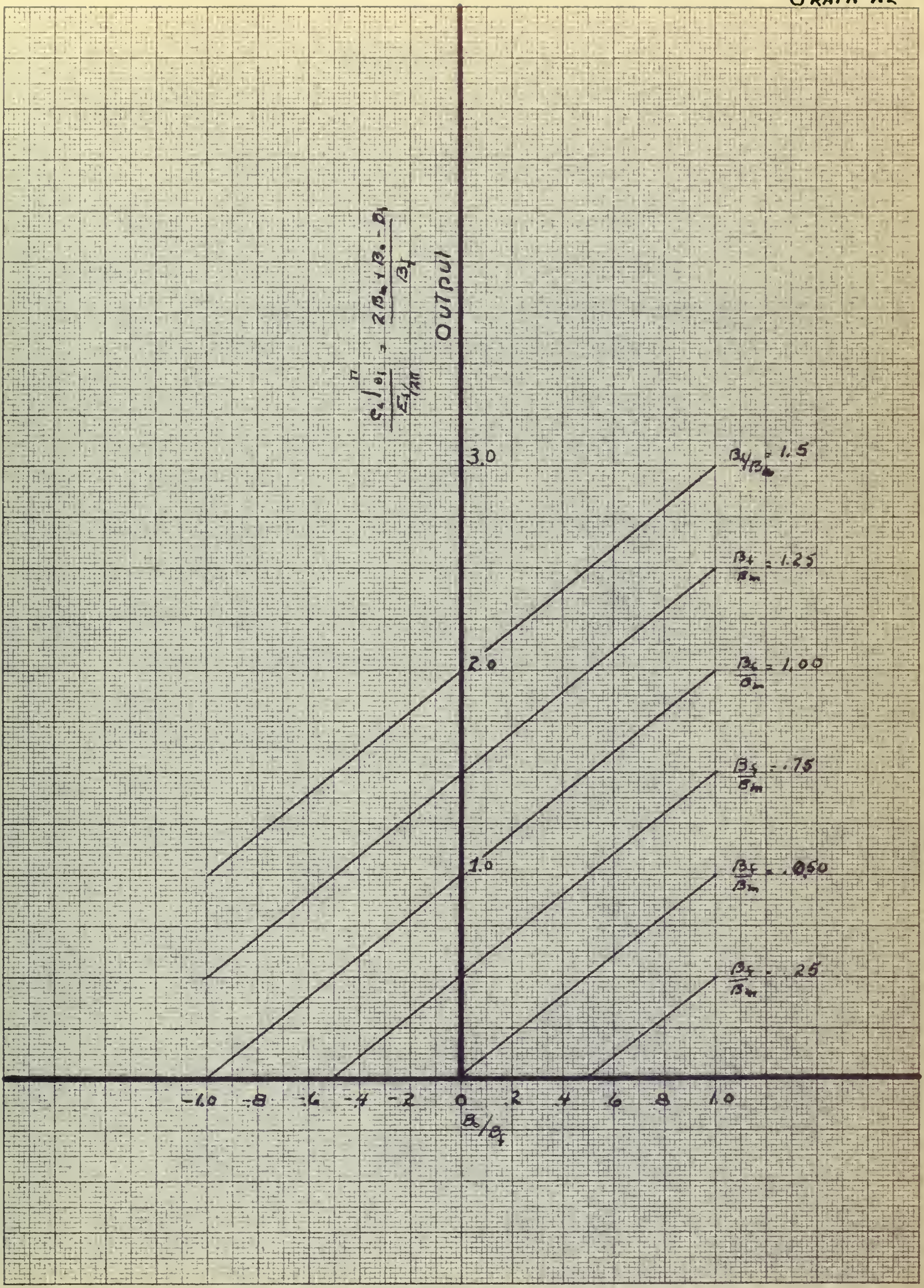
These two graphs are general and may be used so long as approximation #3 is valid. Also for the graph to apply assumptions #1 and #2, that the rectifier has zero forward resistance and infinite back resistance, must be reasonably well approximated.

Circuit Under Consideration

APPENDIX B




Bm/Bf = 1
Bm/Bf = .5
Bm/Bf = .25
Bm/Bf = 1.5
Bm/Bf = 2.0
180°
160°
140°
120°
100°
80°
60°
40°
20°
0
∠ of Firing - θ
-1.0 -.8 -.6 -.4 -.2 0 .2 .4 .6 .8 1.0
Bo/Bf
Ratio of Initial to Critical Magnetic Flux Density


Output
3.0
2.0
1.0
B_t/B_m = 1.5
B_t/B_m = 1.25
B_t/B_m = 1.00
B_t/B_m = .75
B_t/B_m = .050
B_t/B_m = .25
-1.0 -.8 -.6 -.4 -.2 0 .2 .4 .6 .8 1.0
B_r/B_t

APPENDIX B

Determination of the Instantaneous Coil Currents

Simplifications And Assumptions

1. Steady state conditions exist in the amplifier.
2. Load resistor is replaced by a short circuit.
3. Constant voltage impressed on the control winding.
4. Control winding resistance neglected in part of the solution.
5. Leakage flux neglected, and the flux density is assumed constant throughout the core.
6. Eddy currents and hysteresis losses are neglected permitting use of the dc magnetization curve.
7. Rectifier resistance is assumed a constant value in the forward direction and infinite in the reverse direction.
8. All rectifiers are identical.
9. Both ac coils have an equal number of turns.
10. A sinusoidal variation of flux is assumed.

APPENDIX B

The type circuit is shown in the print. During one half cycle of operation the line current flows in one anode winding and during the next half cycle the current flows in the other anode winding, except for the brief period of commutation when current flows in both the anode windings. The output or line current produces a direct component of current in each anode winding. This component of current contributes to the current in the control winding, hence increasing the sensitivity of the control current.

Assumption #10 assumes the flux varies in a sinusoidal manner. To maintain this sinusoidal variation of flux, when there is current in the control windings, even as well as odd harmonics of the mmf are required.

Applying Kirchoff's Laws to the electric loops gives

$$E_{dc} = 2R_1 i_1 + N_1 \left(\frac{d\varphi_1}{dt} + \frac{d\varphi_2}{dt} \right) \quad (1)$$

$$E_m \cos\omega t = R_2 i_2 + N_2 \frac{d\varphi_1}{dt} + i_2 R_r \quad (2)$$

$$E_m \cos\omega t = R_2 i_3 - N_2 \frac{d\varphi_2}{dt} + i_3 R_r \quad (3)$$

R_r is the forward resistance of the rectifiers as assumed in assumption #

During one cycle of operation the following events occur

i_2 flows (positive conduction period)

i_3 flows (negative conduction period)

i_2 and i_3 flows (commutating conduction period)

With uniform distribution of the flux in the core, the flux density is the total flux divided by the cross sectional area of the core.

$$B_{1c} = B_m \sin \omega t + B_o \tag{4}$$

$$B_{2c} = -B_m \sin \omega t + B_o \tag{5}$$

Where

$$B_m = E_m / \omega N A \tag{6}$$

and B_o is the average flux density due to both the control current and the anode current.

The magnetic field intensities of the core can be represented by the following expressions

$$H = U \sinh(uB) + vB \tag{7}$$

$$H_{1c} = H_o + H_1 \sin \omega t + H_2 \cos 2\omega t + H_3 \sin 3\omega t \ldots$$

$$H_{2c} = H_o - H_1 \sin \omega t + H_2 \cos 2\omega t - H_3 \sin 3\omega t \ldots \tag{8}$$

H_o; H_1; etc. are modified Bessel functions of the first kind. The values of the H's are computed with the aid of the table at the end of the appendix. U, u, and v are constants required to fit the H vs. B curve for the core material using the following equation as an analytical expression of the B vs. H curve.

$$H = U \sinh(uB) + vB$$

H_{1c} and H_{2c} are expressed in ampere turns per inch.

Applying Amperes's Law to the two magnetic circuits gives

$$l H_{1c} = N_1 i_1 + N_2 i_2 \tag{9}$$

$$lH_{2c} = N_1 i_1 - N_2 i_3 \tag{10}$$

Where l is the mean magnetic flux path expressed in inches. During the positive conducting period l_3 is equal to zero and solving equations #9 and #10 by subtracting gives

$$i_2 = \frac{l}{N_2}\left(H_{1c} - H_{2c}\right) \tag{11}$$

$$H_{1c} - H_{2c} = 2H_1 \sin\omega t + 2H_3 \sin 3\omega t \tag{12}$$

$$i_1 = \frac{1}{N_1}\left(lH_{1c} - N_2 i_2\right) \tag{13}$$

$$i_2 = \frac{2l}{N_2}\left(H_1 \sin\omega t + H_3 \sin 3\omega t + H_5 \sin 5\omega t \ldots\right) \tag{14}$$

$$i_1 = \frac{l}{N_1}\left[\left(H_0 + H_2 \cos 2\omega t \ldots\right) - \left(H_1 \sin\omega t + H_3 \sin 3\omega t \ldots\right)\right] \tag{15}$$

For the period of negative conduction l_2 is equal to zero and in a similar manner l_3 and 1 may be solved for giving

$$i_3 = \frac{2l}{N_2}\left[H_1 \sin\omega t + H_3 \sin 3\omega t \ldots\right] \tag{16}$$

$$i_1 = \frac{l}{N_1}\left[\left(H_0 + H_2 \cos 2\omega t \ldots\right) + \left(H_1 \sin\omega t + H_3 \sin 3\omega t \ldots\right)\right] \tag{17}$$

During the period of transition l_2 & l_3 may be conducting simultaneously and equations 1, 2, and 3 are applied.

$$E_{dc} = 2R_1 i_1 + N_1\left(\frac{d\psi_1}{dt} + \frac{d\psi_2}{dt}\right) \tag{1}$$

$$E_m \cos\omega t = R_2 i_2 + R_r i_2 + N\frac{d\psi_1}{dt} \tag{2}$$

$$E_m \cos\omega t = R_2 i_3 + R_r i_3 - N\frac{d\psi_2}{dt} \tag{3}$$

$$E_{dc} N_2 = 2 R_1 N_2 i_1 + N_1 N_2 \left(\frac{d}{dt} (\psi_1 + \psi_2) \right) \tag{1a}$$

Subtracting equation #2 from #3

$$0 = (R_2 + R_r)(-i_2 + i_3) N_1 - N_1 N_2 \frac{d(\psi_1 + \psi_2)}{dt} \qquad (-2 \quad 3)$$

Adding equation (1a) to equation #3 and subtracting equation #2 gives

$$E_{dc} N_2 = 2 R_1 N_2 i_1 + N_1 (-i_2 + i_3)(R_2 + R_r) \tag{18}$$

$$-i_2 = \frac{N_1 i_1 - \ell H_{1c}}{N_2} \tag{9a}$$

$$i_3 = \frac{N_1 i_1 - \ell H_{2c}}{N_3} \tag{10a}$$

$$E_{dc} N_2 = 2 R_1 N_2 i_1 + N_1 (R_2 + R_r) \left[\frac{2 N_1 i_1 - \ell (H_{1c} + H_{2c})}{N_2} \right] \tag{19}$$

$$\frac{E_{dc}}{2R_1} = i_1 + \left(\frac{N_1}{N_2}\right)^2 \left(\frac{R_2 + R_r}{R_1}\right) i_1 - \frac{(R_2 + R_r)}{2 N_2^2 R_1} \ell N_1 (H_{1c} + H_{2c}) \tag{20}$$

$$\text{Let } k' = \left(\frac{N_1}{N_2}\right)^2 \left(\frac{R_2 + R_r}{R_1}\right) \tag{21}$$

$$I_1 = \frac{E_{dc}}{2 R_1} \tag{22}$$

$$I_1 = (1 + k') i_1 - \frac{\ell k'}{2 N_1} \left(H_{1c} + H_{2c} \right) \tag{23}$$

$$i_1 = \frac{1}{(1+k')} \left[\frac{k' \ell}{2 N_1} (H_{1c} + H_{2c}) + I_1 \right] \tag{24}$$

Substituting for H_{1c} and H_{2c}

$$i_1 = \frac{1}{(1+k')} \left[\frac{k' \ell}{N_1} (H_0 + H_2 \cos 2\omega t + H_4 \cos 4\omega t \ldots) + I_1 \right] \tag{25}$$

$$i_3 = \frac{N_1 i_1 - \ell H_{2c}}{N_2} \tag{26}$$

$$i_3 = \frac{1}{1+k'}\left[k'\ell(H_0 + H_2\cos 2\omega t + H_4\cos 4\omega t \ldots) + N_1 I_1\right]$$

$$- \frac{\ell}{N_2}\left[H_0 - H_1\sin\omega t + H_3\sin 3\omega t \ldots\right] \qquad (27)$$

$$i_3 = \frac{\ell}{N_2}\left[H_1\sin\omega t + H_3\sin 3\omega t \ldots\right]$$

$$- \frac{1}{N_2(1+k')}\left[\ell(H_0 + H_2\cos 2\omega t \ldots) - N_1 I_1\right] \qquad (28)$$

Similarly l_2 is solved for and gives the following form;

$$i_2 = \frac{\ell}{N_2}\left[H_1\sin\omega t + H_3\sin 3\omega t \ldots\right]$$

$$+ \frac{1}{(1+k')N_2}\left[\ell(H_0 + H_2\cos 2\omega t \ldots) - N_1 I_1\right] \qquad (29)$$

$$i_2 + i_3 = \frac{2\ell}{N_2}\left[H_1\sin\omega t + H_3\sin 3\omega t \ldots\right] \qquad (30)$$

The sum of l_2 and l_3 is equal to zero when

Rearranging equations 1, 2, and 3 for the instant l_3 is equal to zero

$$E_{dc} = 2R_1 i_1 + N_1\left[\frac{d(\psi_{1c} + \psi_{2c})}{dt}\right] \qquad (1)$$

$$E_m\cos\omega t = R_2 i_2 + N_2\frac{d\psi_{1c}}{dt} + i_2 R_r \qquad (2)$$

$$E_m\cos\omega t = -\frac{N_2\, d\psi_{2c}}{dt} \qquad (3a)$$

$$N_2 E_{dc} = 2R_1 i_1 N_2 - i_2(R_2 + R_r)N_1 \qquad (31)$$

$$\frac{E_{dc}}{2R_1} = i_1 - \frac{N_1^2}{2N_2}\left(\frac{R_2 + R_r}{R_1}\right)i_2 \qquad (32)$$

$$N_1 I_1 = N_1 i_1 - \frac{N_1^2}{2N_2}\left(\frac{R_2 + R_r}{R_1}\right)i_2 \qquad (33)$$

Replace l_1 and l_2 from the values during the positive

half cycle;

$$N_1 I_1 = \ell\left[(H_0 + H_2\cos 2\omega t\ldots) - (H_1 \sin\omega t + H_3 \sin 3\omega t\ldots)\right]$$

$$- \frac{N_1^2}{2N_2^2}\, 2\ell\left[(H_1 \sin\omega t + H_3 \sin 3\omega t\ldots)\right]\left(\frac{R_2 + R_r}{R_1}\right) \qquad (34)$$

$$= \ell\left[(H_0 + H_2\cos 2\omega t\ldots) - (1+k')(H_1 \sin\omega t + H_3 \sin 3\omega t\ldots)\right] \qquad (35)$$

Let $\omega t = \sigma$ be the instant 1_3 begins to conduct

$$N_1 I_1 = \ell\left[(H_0 + H_2\cos\sigma\ldots) - (1+k')(H_1 \sin\sigma + \frac{H_3}{3}\sin 3\sigma\ldots)\right] \qquad (36)$$

the time average value of 1_1 is;

$$i_{1(ave)} = \frac{1}{2\pi}\int_0^{2\pi} i_1\, d(\omega t) \qquad (37)$$

Using the expression for 1_1 during the three successive stages and the proper limits of integration for each gives;

$$N_1 I_1 = \ell H_0 - \frac{2\ell}{\pi(1+k') - 2\sigma}\left\{\left[\frac{H_2}{2}\sin 2\sigma + \frac{H_4}{4}\sin 4\sigma\ldots\right] + (1+k')\left[H_1\cos\sigma + \frac{H_3}{3}\cos 3\sigma\ldots\right]\right\} \qquad (38)$$

Equating equation #39 and #41

$$\ell\left[(H_0 + H_2\cos 2\sigma\ldots) + (1+k')(H_1 \sin\sigma + H_3 \sin 3\sigma\ldots)\right] =$$

$$\ell H_0 - \frac{2\ell}{\pi(1+k') - 2\sigma}\left[\left(\frac{H_2}{2}\sin 2\sigma + \frac{H_4}{4}\sin 4\sigma\ldots\right) + (1+k')(H_1\cos\sigma + \frac{H_3}{3}\cos 3\sigma\ldots)\right] \qquad 39)$$

$$(H_2\cos 2\sigma + H_4\cos 4\sigma\ldots) - (1+k')(H_1\sin\sigma + H_3\sin 3\sigma\ldots) =$$

$$\frac{-2(1+k')}{\pi(1+k') - 2\sigma}\left[(H_1\cos\sigma + \frac{H_3}{3}\cos 3\sigma\ldots) + \left(\frac{H_2}{2}\sin 2\sigma + \frac{H_4}{4}\sin 4\sigma\ldots\right)\right] \qquad (40)$$

If terms higher than the second harmonic are omitted then the expression can be reduced to

$$\tan\sigma - \frac{1}{\frac{\pi}{2}(1+k') - \sigma} = \frac{1}{\tan\sigma + \frac{H_{1c}}{H_{2c}}(1+k')\sec\sigma} \tag{41}$$

TABLE ONE

$$H_0 = U\, I_0(uB_m)\sinh uB_0 \quad + uB_0$$

$$H_1 = 2U I_1(uB_m)\cosh uB_0 \quad + uB_0$$

$$H_2 = -2U I_2(uB_m)\sinh uB_0$$

$$H_3 = -2U I_3(uB_m)\cosh uB_0$$

$$H_4 = 2U I_4(uB_m)\sinh uB_0$$

$$H_5 = 2U I_5(uB_m)\cosh uB_0$$

APPENDIX C

Universal Curves for Magnetic Cores

Simplifying Assumptions

1. Sinusoidal variation of flux with an impressed sinusoidal voltage.

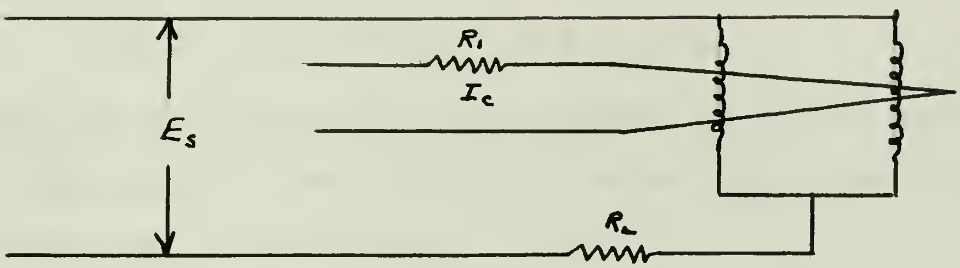
2. Assume the flux follows the normal magnetization curve when the core is unsymmetrically magnetized.

3. Neglect the core loss.

4. The d.c. magnetization curve can be approximated by the following expression

$$i_2 = C_1 U + C_N U^N$$

5. Uniform flux distribution in the core.



If there is no current in the control winding then from the first simplifying assumption, the expression for the instantaneous flux is represented as follows:

$$U = U_m \sin \omega t \qquad (1)$$

$$U_m = B_m A \qquad (2)$$

Example - Let n = 9 then

$$i_2 = C_1 U_m \sin \omega t + C_9 U_m^9 \sin^9 \omega t \tag{3}$$

The rms value of the current is the square root of the sum of the squares.

$$I_2^2 = \frac{2}{\pi}\left[C_1^2 U_m^2 \int_0^{\frac{\pi}{2}} \sin^2 \omega t \, d(\omega t) + C_1 C_9 U_m^{10} \int_0^{\pi/2} \sin^{10} \omega t \, d(\omega t) + C_9 U_m^{18} \int_0^{\frac{\pi}{2}} \sin 18 \, \omega t \, d(\omega t)\right] \tag{4}$$

Using B. O. Pierce's book - "A Short Table of Integrals" equation intergrades into

$$I_2^2 = \frac{C_1^2 U_m^2}{2} + \frac{C_1 C_9 U_m^{10}}{(\pi)^{1/2}} \frac{\Gamma(5.5)}{\Gamma(6)} + C_9^2 U_m^{18} \frac{\Gamma(9.5)}{\Gamma(10)} \tag{5}$$

An approximate value of this expression is

$$I_2 = \frac{C_1 U_m}{(2)^{1/2}} + C_9 U_m^9 \left(\frac{\Gamma(9.5)}{\Gamma(10)}\right)^{1/2} \tag{6}$$

$$I_2 \doteq 0.707\, C_1 U_m + 0.43\, C_9 U_m^9 \tag{7}$$

This is the rms value of the current in one of the anode coils. The line or load current is twice this value for the coils connected in parallel.

$$I_{2+3} = 1.414\, C_1 U_m + .86\, C_9 U_m^9 \tag{8}$$

The equation under assumption #4 or equation #8 permits the calculation of the instantaneous or the rms value of the anode current, for the case when the control current is zero.

With dc control current U becomes

$$U = U_0 + U_m \sin \omega t \tag{9}$$

Substituting this in the equation of assumption #4 and

letting n = 9.

$$i_2 = C_1(U_0 + U_m \sin\omega t) + C_9(U_0 + U_m \sin\omega t)^9 \qquad (10)$$

Expanding the last term by the binomial theorem

$$i_2 = C_1U_0 + C_1U_m \sin\omega t + C_9\Big[U_0^9 + 9U_0^8U_m \sin\omega t$$

$$+ 36U_0^7U_m^2(.5 - .5\cos 2\omega t) + 84U_0^6U_m^3(.75\sin\omega t - .25\sin 3\omega t)$$

$$+ 126U_0^5U_m^4(.375 - .5\cos 2\omega t + .125\cos 4\omega t) + 126U_0^4U_m^5(.625\sin\omega t$$

$$- .313\sin 3\omega t + .063\sin 5\omega t) + 84U_0^3U_m^6(.313 - .469\cos 2\omega t$$

$$+ .188\cos 4\omega t - .031\cos 6\omega t) + 36U_0^2U_m^8(.547\sin\omega t - .328\sin 3\omega t$$

$$.109\sin 5\omega t - .016\sin^7\omega t) + 9U_0U_m^8(.273 - .436\cos 2\omega t + .219\cos 4\omega t$$

$$+ .016\cos 6\omega t + .008\cos 8\omega t) + U_m^9(.492\sin\omega t - .328\sin 3\omega t$$

$$+ .141\sin 5\omega t - .004\sin 7\omega t + .004\sin 9\omega t) \qquad (11)$$

In the A.C. line current the odd harmonics in each leg add but the even harmonics circulate between the windings and do not appear in the line circuit. The instantaneous line current is

$$i_T = \hat{I}_1 \sin\omega t - \hat{I}_3 \sin 3\omega t \;\; . \; . \; . \;\; I_9 \sin 9\omega t. \qquad (12)$$

$$\hat{I}_1 = 2C_1U_m + 2C_9\left[9U_0^8U_m + 63U_0^6U_m^3 + 78.7U_0^4U_m^5 + 19.7U_0^2U_m^7 + .492U_m^9\right]$$

$$\hat{I}_3 = 2C_9\left[21U_0^6U_m^3 + 39.4U_0^4U_m^5 + 11.8U_0^2U_m^7 + .328U_m^9\right]$$

$$\hat{I}_5 = 2C_9\left[7.87U_0^4U_m^5 + 3.9U_0^2U_m^7 + .141U_m^9\right]$$

$$\hat{I}_7 = 2C_9\left[.562U_0U_m^7 + .035U_m^9\right]$$

$$\hat{I}_9 = 2C_9\left[.004U_m^9\right]$$

Dividing both sides by

$$\Delta_m = \frac{\hat{I}_n}{2C_9 U_m^9} \; ; \qquad a = \frac{C_1}{C_9 U_m^8} \; ; \qquad r = \frac{U_0}{U_m}$$

Then

$$\Delta_1 = a + 9r^8 + 63r^6 + 78.7r^4 + 19.7r^2 + .492 \tag{13}$$

$$\Delta_3 = 21r^6 + 39.4r^4 + 11.8r^2 + .328$$

$$\Delta_5 = 7.87r^4 + 3.94r^2 + .141$$

$$\Delta_7 = .562r^2 + .035$$

$$\Delta_9 = .004$$

$$\Delta_{rms} = \sqrt{\frac{\Sigma \Delta_n^2}{2}} = \frac{I_T}{2C_9 U_m^9}$$

For r equal to one tenth:

$$\Delta_1^2 = (a + .69)^2 \tag{14}$$

$$\Delta_3^2 = (.47)^2 = .22$$

$$\Delta_5^2 \; ; \; \Delta_7^2 \; ; \; \Delta_9^2 \ll \Delta_3^2$$

$$\Sigma \Delta_{rms} = a + .87$$

$$\frac{I_T}{2C_9 U_m^9} = \frac{\Sigma \Delta_{rms}}{\sqrt{2}} = a + .87 \tag{15}$$

$$I_T = 1.414\, C_1 U_m + 1.23\, C_9 U_m^9 \tag{16}$$

Define

$$I_{rms} = I_T - 1.414\, C_1 U_m \tag{17}$$

$$I_{rms} = 1.23\ C_9\ U_m^9 \qquad (18)$$

This is the curved portion of the magnetization curve less the straight line portion of the curve. Designating I'_{rms} as the current at the point of fit, and U'_m as the peak value of the a.c. flux at the point of fit.

$$I'_{rms} = I'_T - 1.414\ C_1 U_m \qquad (19)$$

Substituting these values into equation #8

$$I'_{rms} = .86\ C_9 U'^9_m + 1.414\ C_1 U_m - 1.414\ C_1 U_m \qquad (20)$$

$$I'_{RMS} = .86\ C_9\ U'^9_m \qquad (21)$$

Dividing equation #18 by #21

$$\frac{I_{RMS}}{I'_{RMS}} = I_{RPU} = \frac{1.23}{.86}\ \frac{C_9\ U_m^9}{C_9\ U'^9_m} \qquad (22)$$

$$\frac{U_m}{U'_m} = \frac{E_{rms}}{E'_{rms}} = E_{rpu} \qquad (23)$$

$$I_{rpu} = 1.42\ E_{rpu}^9 \qquad (24)$$

Similar equations can be developed for values of r other than 0.1. Some of these are listed below

r	Equations	(25)
.1	$I_{rpu} = 1.42\ E_{rpu}^9$	
.5	$I_{rpu} = 2.14\ E_{rpu}^9$	
1.0	$I_{rpu} = 3.05 \times 10^2 E_{rpu}^9$	

r	
3.0	$I_{rpu} = 1.87 \times 10^{5} E_{rpu}^{9}$
4.0	$I_{rpu} = 1.44 \times 10^{6} E_{rpu}^{9}$

These curves are shown on graph #1c for different values of r. The constant term in equation "11" is the d.c. current in the control winding required to produce the different degrees of saturation. Listed separately they are:

$$I_c = C_1 U_o + C_9 \left[U_o^9 + 18 U_o^7 U_m^2 + 47.2 U_o^5 U_m^4 + 26.2 U_o^3 U_m^6 + 2.46 U_o U_m^8 \right] \qquad (26)$$

$$\frac{I_o}{C_9 U_m^9} = \frac{I_c - C_1 U_o}{C_9 U_m^9} = r^9 + 18r^7 + 42.2r^5 + 26.2r^3 + 2.46r \qquad (27)$$

$$I_o = C_9 U_m^9 (r^9 + 18r^7 + 42.2r^5 + 26.2r^3 + 2.46r) \qquad (28)$$

For various values of r

r	I_o
0	$I_o = 0$
.1	$I_o = 2.72 \times 10^{-1} C_9 U_m^9$
.5	$I_o = 6.14\ C_9 U_m^9$
1.0	$I_o = 9.48 \times 10\ C_9 U_m^9$
3.0	$I_o = 7.12 \times 10^{4} C_9 U_m^9$
4.0	$I_o = 6.06 \times 10^{5} C_9 U_m^9$

(29)

Dividing the equations listed under #29 by equation #21:

$$\frac{I_o}{I'_{rms}} = I_{opu} = (r^9 + 18r^7 + 42.2r^5 + 26.2r^3 + 2.46r) \frac{C_9 U_m^9}{.86 C_9 U_m^9} \qquad (30)$$

r	
0	$I_{opu} = 0$
.1	$I_{opu} = 3.16 \times 10^{-1} E_{rpu}^{9}$
.5	$I_{opu} = 7.15 \; E_{rpu}^{9}$
1.0	$I_{opu} = 1.10 \times 10^{2} E_{rpu}^{9}$
3.0	$I_{opu} = 8.28 \times 10^{4} E_{rpu}^{9}$
4.0	$I_{opu} = 7.05 \times 10^{5} E_{rpu}^{9}$

(31)

Choosing values of I_{opu} as a parameter, E_{rpu} may be solved for using r as the variable.

I_{opu}	r	E_{rpu}	I_{rpu}
0.05	.1	1.58 x 10	0.221
0.05	.5	6.99 x 10	0.150
0.05	1.0	4.52 x 10	0.138
0.05	3.0	6.04 x 10	0.113
0.05	4.0	7.08 x 10	0.102
0.1	.1	3.16 x 10	0.443
0.1	.5	1.40 x 10	0.289
0.1	1.0	9.04 x 10	0.276
0.1	3.0	1.21 x 10	0.226
0.1	4.0	1.42 x 10	0.204
0.15	.1	4.74 x 10	0.664
0.15	.5	2.10 x 10	0.449
0.15	1.0	1.36 x 10	0.414
0.15	3.0	1.81 x 10	0.339
0.15	4.0	2.12 x 10	0.306
0.20	.1	6.32 x 10	0.886
0.2	.5	2.80 x 10	0.598
0.2	1.0	1.81 x 10	0.552
0.2	3.0	2.42 x 10	0.452
0.2	4.0	2.83 x 10	0.408
0.3	.1	9.48 x 10	1.33
0.3	.5	9.14 x 10	0.897
0.3	1.0	2.72 x 10	0.828
0.3	3.0	3.63 x 10	0.678
0.3	4.0	4.25 x 10	0.612

This information is plotted on graph #1C. The next value of current that is important is the average value of the anode current since in most applications it is this value of current that is used with feedback circuits. Without d.c. saturation

$$I_{ave} = \frac{C_1 U_m}{\pi}\int_0^{\frac{\pi}{2}} \sin \omega t \, d(\omega t) + \frac{2C_9 U_m^9}{\pi}\int_0^{\frac{\pi}{2}} \sin^9 \omega t \, d(\omega t) \quad (32)$$

The average value of equation #32 is given next.

$$I_{(AVE)1} = 0.636\, C_1 U_m + .257\, C_9 U_m^9 \quad (33)$$

The total average current for the two anode coils in parallel is twice this value.

$$I_{(AVE)T} = 1.27\, C_1 U_m + .514\, C_9 U_m^9 \quad (34)$$

With a current in the control windings, from equation #12

$$i_t = \hat{I}_1 \sin \omega t - \hat{I}_3 \sin 3\omega t \ldots \hat{I}_9 \sin 9\omega t \quad (35)$$

This current can be averaged as before for the rms values.

$$I_{(AVE)T} = \left[.636\hat{I}_1 + .127\hat{I}_5 + .074\hat{I}_9 - .212\hat{I}_3 - .091\hat{I}_7\right] \quad (36)$$

$$\Delta_{AVE} = \frac{I_{(AVE)T}}{2C_9 U_m} = \left[.631\Delta_1 - .212\Delta_3 + .127\Delta_5 - .091\Delta_7 + .074\Delta_9\right] \quad (37)$$

Computing the equation of $I_{T(ave)}$ for the value of r equal to 0.1:

$$\Delta_{AVE} = .636a + .365\,, \qquad a = \frac{C_1}{C_9 U_m^8} \quad (38)(39)$$

$$I_{T(AVE)} = 1.272\, C_1 U_m + .73\, C_9 U_m^9 \qquad (40)$$

Defining $I_{(ave)}$ by the following equation

$$I_{T(AVE)} = I_{AVE} + 1.272\, C_1 U_m \qquad (41)$$

$$I_{apu} = \frac{I_{T(AVE)}}{I'_{RMS}} = \frac{.73\, C_9 U_m^9}{.86\, C_9 U_m^9} = 0.85\, E_{rpu}^9 \qquad (42)$$

for the value of r equal to 0.1:

r	I_{apu}
0.1	$I_{apu} = .85 \times 10^{-1} E_{rpu}^9$
0.5	$I_{apu} = 1.43 \times 10\ E_{rpu}^9$
1.0	$I_{apu} = 2.70 \times 10^2 E_{rpu}^9$
3.0	$I_{apu} = 1.56 \times 10^5 E_{rpu}^9$
4.0	$I_{apu} = 1.24 \times 10^6 E_{rpu}^9$
0	$I_{apu} = .598\ E_{rpu}^9$

(43)

From the table following equation #31 the values of E^9_{rpu} are available for substituting into equations #43 for the evaluation of I_{apu}. The following table tabulates these results.

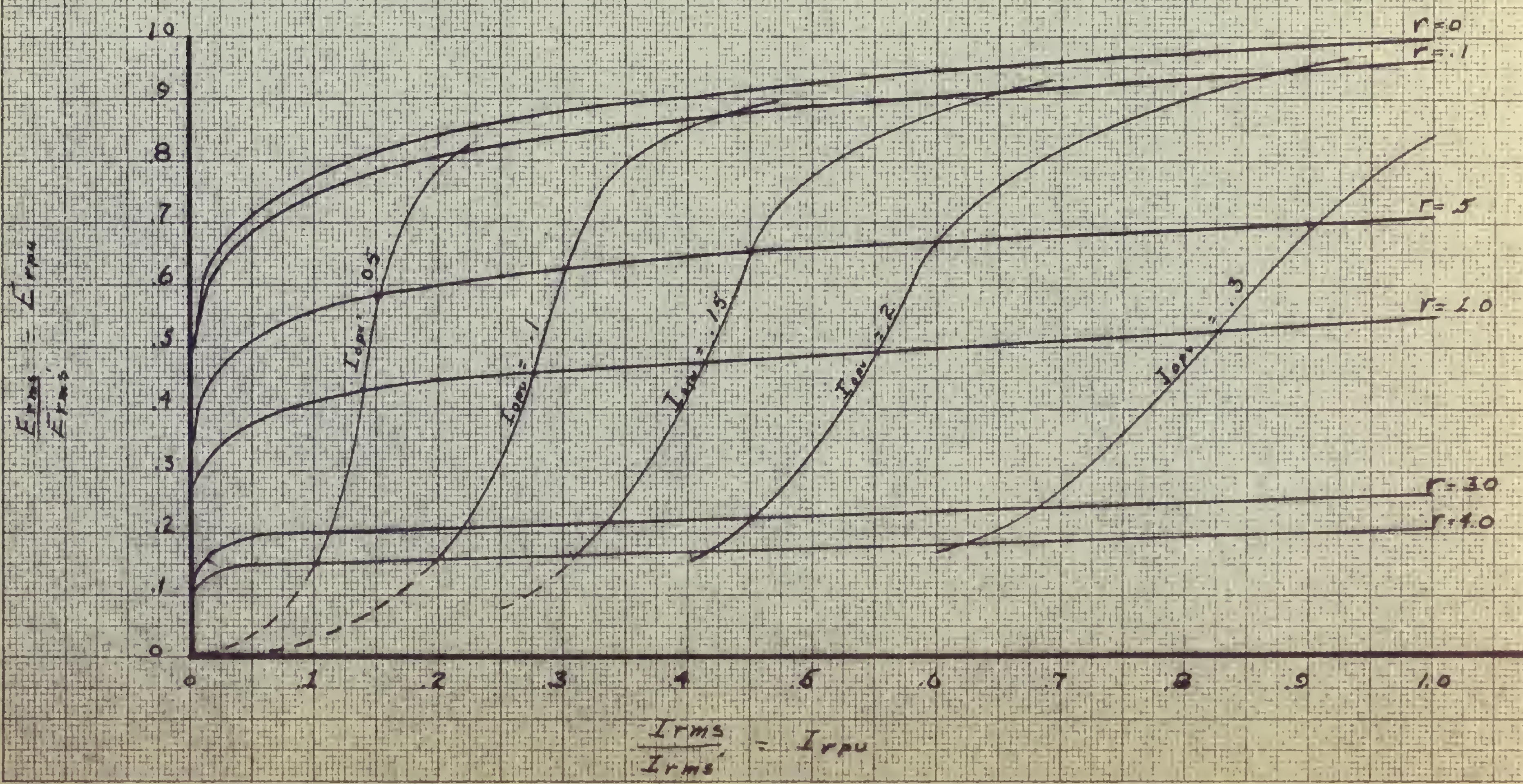
I_{opu}	r	I_{apu}	I_{opu}	r	I_{apu}
0.05	.1	.135	.1	.1	.27
0.05	.5	.1	.1	.5	.20
0.05	1.0	.1	.1	1.0	.20
0.05	3.0	.095	.1	3.0	.19
0.05	4.0	.095	.1	4.0	.176
0.15	.1	.405	.2	.1	.54
0.15	.5	.30	.2	.5	.4
0.15	1.0	.30	.2	1.0	.4
0.15	3.0	.285	.2	3.0	.38
0.15	4.0	.264	.2	4.0	.352
0.3	.1	.81			
0.3	.5	.60			
0.3	1.0	.60			
0.3	3.0	.57			
0.3	4.0	.528			

These results are graphed on graph #2C. Note here as in Appendix A the graphs are so constructed that the unknown factors that vary with materials are divided out. The results are that the curves can be used with cores with different magnetizing characteristics. Other graphs other than the ninth power can also be computed and graphed. The fifth, ninth, and fifteenth power curves are shown on graphs #1 and #2 in Chapter two.

GRAPH 1C

$\frac{E_{rms}}{E_{rms}'}$ vs $\frac{I_{rms}}{I_{rms}'}$ for different values of $r = \frac{U_o}{U_m}$

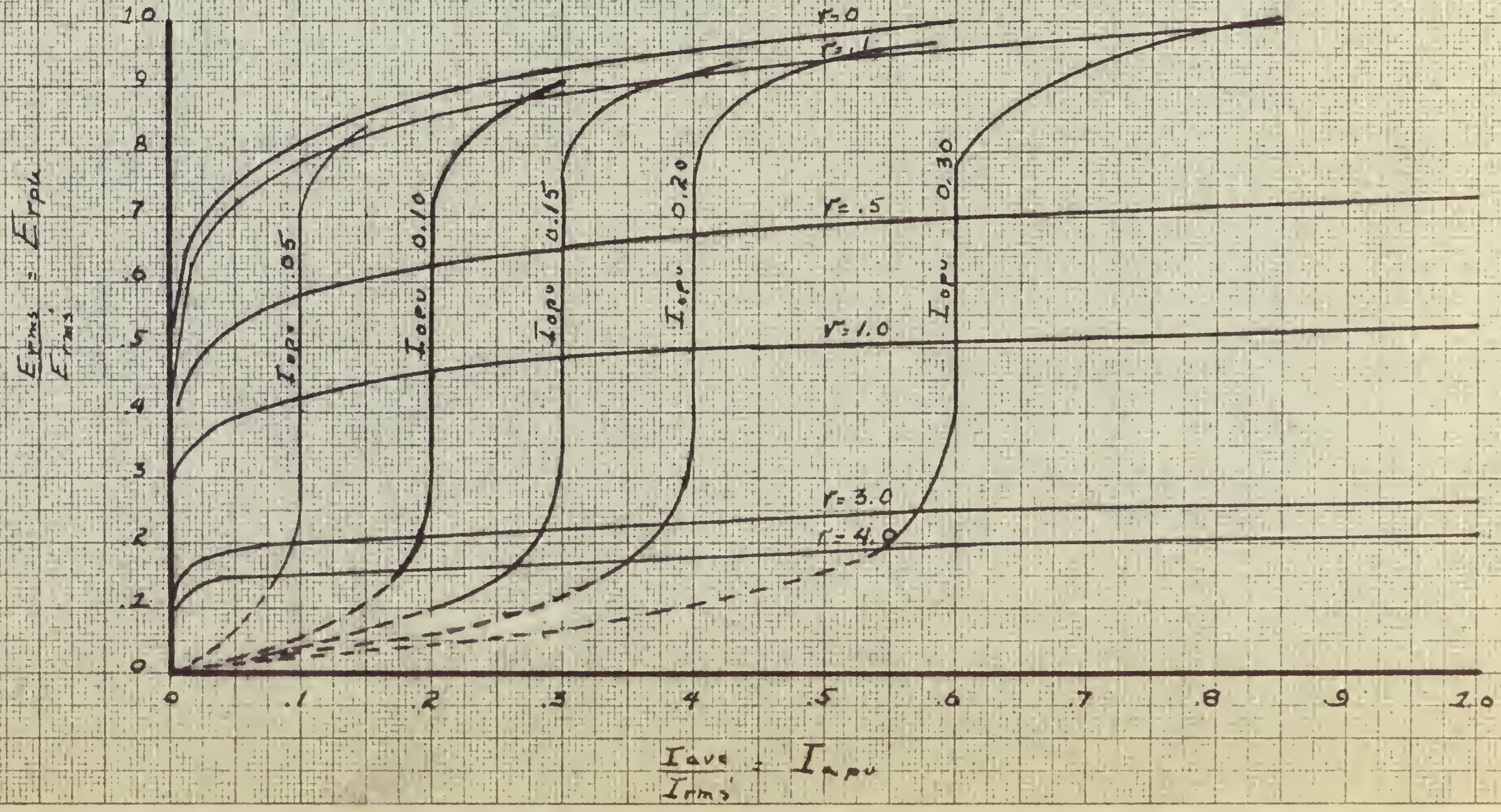
and $I_{opu} = I_o N_{ac} / I_{rms}' N_{ac}$ WITH a NINETH POWER SATURATION CURVE



GRAPH 2C

$\frac{E_{rms}}{E_{rms}'}$ vs $\frac{I_{ave}}{I_{rms}'}$ for different values of $r = \frac{U_o}{U_m}$

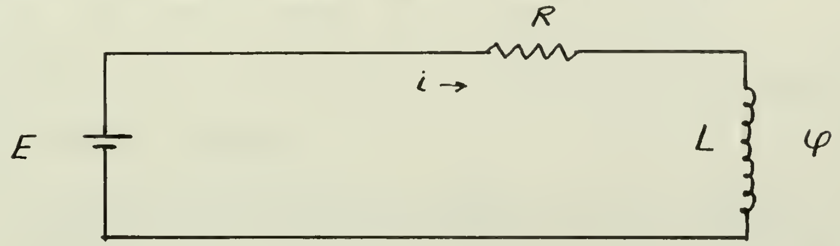
and $I_{opu} = I_o N_{ac} / I_{rms}' N_{ac}$ FOR A NINETH POWER SATURATION CURVE



APPENDIX D

Magnetic Response Determination by Graphical Method

The following equivalent circuit may be used for determination of the response time of the amplifier:



$$\frac{d\varphi}{dt} + iR = E \qquad (1)$$

Where the total change of flux is a nonlinear function of time due to the changing value of L as changes. The method of taking increments is performed as follows:

$$\frac{\Delta\varphi}{\Delta t} = E - iR \qquad (2)$$

$$\frac{\Delta\varphi}{\frac{E}{R} - i} = R\Delta t \qquad (3)$$

Graph 1-D of φ vs. I is a graph of equation #1. For convenience of computing make the construction as follows: Construct a vertical line at I = E/R and another at I = 2E/R. Choose a point "P" on the curve, and a $\Delta\varphi$ with "P" as its center. Draw a line "DCB" as shown on the graph #1-D. Next draw a line AF from the other end of $\Delta\varphi$ and passing through "C". The two triangles ABC and CDF are similar.

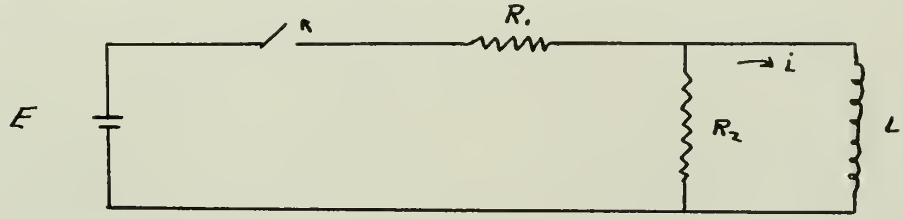
$$\frac{\Delta\varphi}{\frac{E}{R} - i} = \frac{DF}{E/R} \qquad (4)$$

$$\text{Let } DF = h; \quad \frac{\Delta\varphi}{\frac{E}{R} - i} = R\Delta t = \frac{h}{\frac{E}{R}} \qquad (5)$$

$$\Delta t = \frac{h}{E} \qquad (6)$$

Thus a current change Δi causes a flux change $\Delta\varphi$ in a time $\Delta t = h/E$. Δt will be in seconds, $\Delta\varphi$ is in webers and E is in volts. If a large $\Delta\varphi$ is to be considered it should be broken up into several increments for accurate results. The total time is the sum of each incremental time.

To calculate the decay time the equivalent circuit is rearranged as shown below:



$$-\frac{d\varphi}{dt} + R_2 i = 0 \qquad (7)$$

$$\frac{\Delta\varphi}{i} = R_2 \Delta t \qquad (8)$$

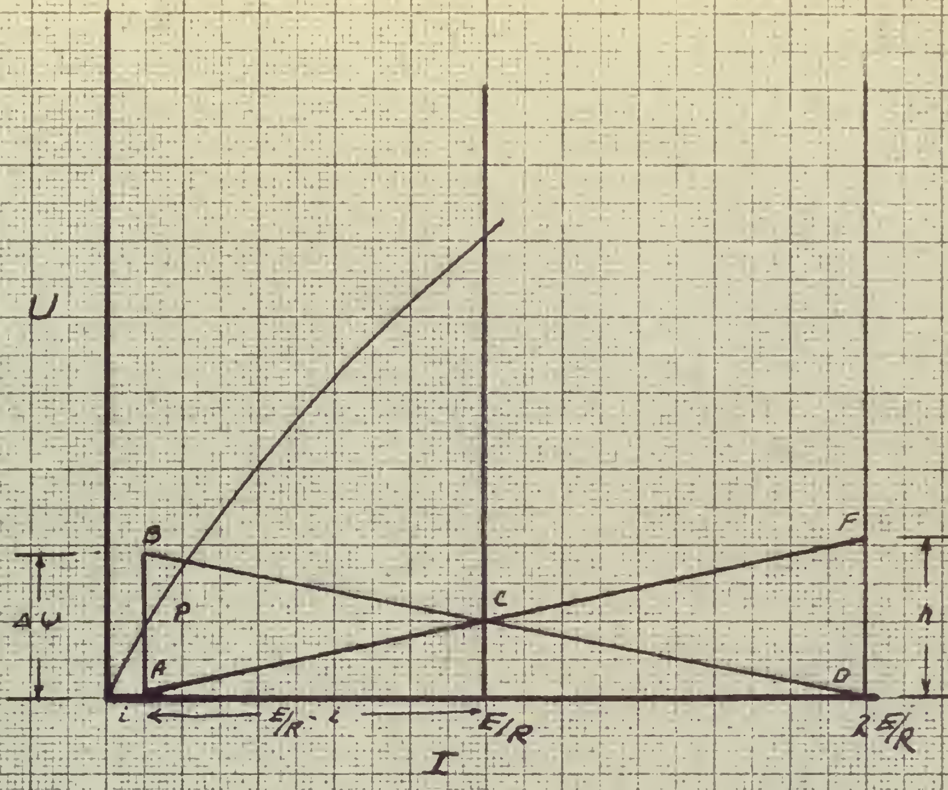
R_z is the equivalent reflected resistance from the anode winding and was included in the R_1 of the equivalent circuit for the computation of the time of rise. The method of construction is shown in graph #2-D. k is equal to -E/R.

$$\frac{\Delta\varphi}{i} = \frac{-m}{-k} = R_2 \Delta t \qquad (9)$$

$$\Delta t = \frac{m}{k R_2} \qquad (10)$$

The time for a large change of $\Delta\varphi$ is computed in a manner similar to a large rise by taking several increments.

GRAPH 1-D



GRAPH 2-D